

Ch. 0 6 Basic Constructions

Construction 1

Given a segment, construct a congruent segment.

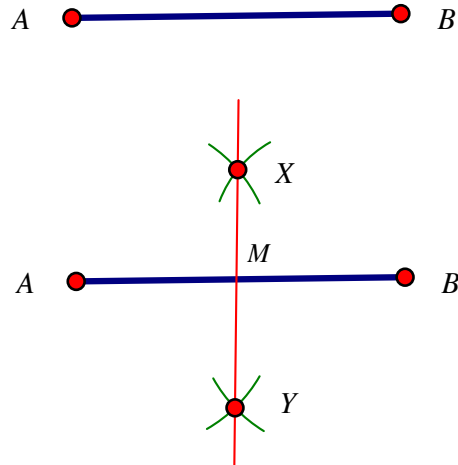


Steps

1. Draw a line \overline{AB}
2. Using a compass with X as the center, construct an arc intersecting \overline{XY} at Y
3. On the line \overline{AB} , using the compass with A as the center, construct the same arc and label the point of intersection B.

Construction 2

Given a segment, construct a perpendicular bisector.



Steps

1. Using a compass with a radius greater than $\frac{1}{2} \overline{AB}$ construct two arcs (above & below line segment) having A as center and two arcs having B as center
2. Label the points of intersection X and Y
3. Draw line \overline{XY}

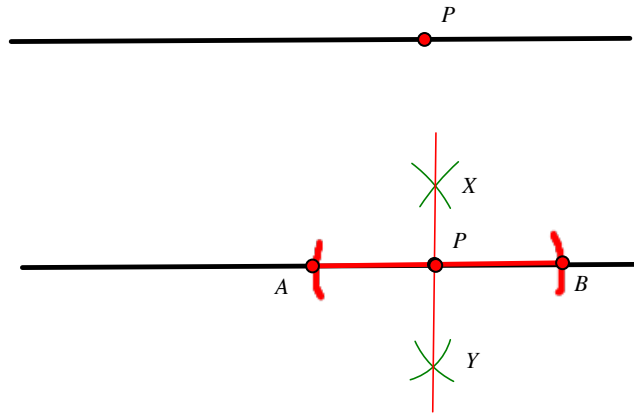
Justification

$\triangle AXY \cong \triangle BXY$ and $\triangle AXB \cong \triangle AYB$ by SSS, then $\angle XMB \cong \angle XMA$ and $\overline{AM} \cong \overline{MB}$. $\angle XMB$ and $\angle XMA$ are congruent adjacent angles, therefore they form perpendicular lines and since $\overline{AM} \cong \overline{MB}$, M is the midpoint so by definition a bisector,

The next two constructions are based on the previous construction of a perpendicular bisector of a segment. What we do in the following two constructions is make a new line segment using the intersection of the arcs and the given line segment, then construct a perpendicular bisector as we just did.

Construction 3

Given a point on a line, construct a line perpendicular to the line through that point.



Steps

1. Using P as the center, construct two arcs, one on each side of P. Label the points of intersection as A and B.

Now use the steps for constructing a \perp bisector

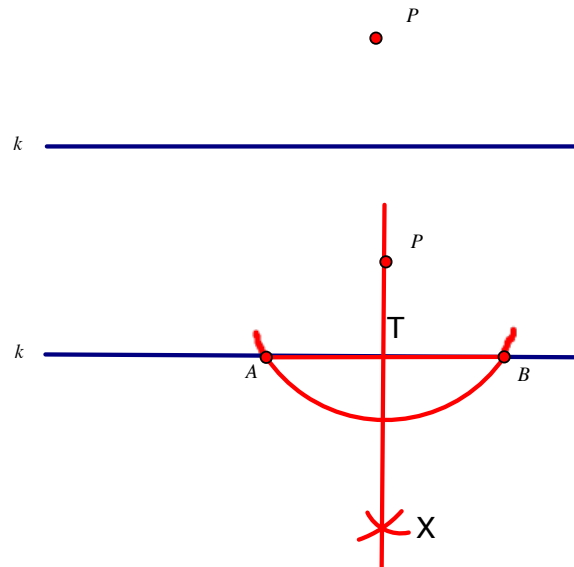
2. Using a compass with a radius greater than $1/2 \overline{AB}$ construct two arcs (above & below line segment) having A as center and two arcs having B as center
3. Label the points of intersection X and Y
4. Draw line \overline{XY}

Justification

When \overline{AX} and \overline{BX} are drawn, then $\triangle APX \cong \triangle BPX$ by SSS. By cpctc, $\angle APX \cong \angle BPX$, the lines are perpendicular because they form congruent adjacent angles.

Construction 4

Given a point not on the line, construct a perpendicular to the line through the point.



Steps

1. Using P as the center, construct an arc intersecting k at two points. Label the points A and B

Now use the steps for constructing a \perp bisector

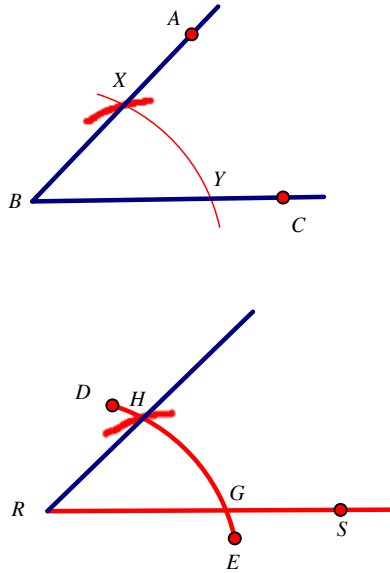
2. Using a compass with a radius greater than $1/2 \overline{AB}$ construct two arcs (below line segment) having A as center and two arcs having B as center
3. Connect the intersection of the arcs X to P

Justification

Draw \overline{PA} , \overline{PB} , \overline{XA} , and \overline{XB} . $\overline{PA} \cong \overline{PB}$ and $\overline{XA} \cong \overline{XB}$, so $\triangle PAX \cong \triangle PBX$ by SSS. $\angle PAT \cong \angle PBT$, base angles of isosceles triangle, $\angle APT \cong \angle BPT$ - cpctc. $\triangle APT \cong \triangle BPT$ by ASA. $\angle PTA \cong \angle PTB$, cpctc which are congruent adjacent angles, therefore the lines are perpendicular

Construction 5

Given an angle, construct a congruent angle.



Steps

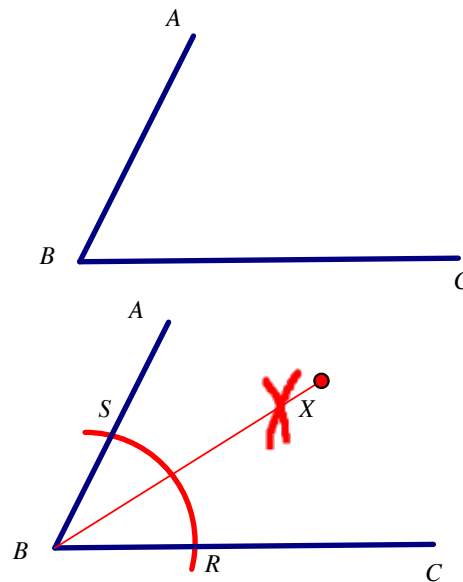
1. Draw ray RS
2. Using the compass with B as the center, make an arc intersecting the rays and label the intersections X and Y.
3. With the same radius, draw the same arc with R as the center, label the point of intersection G.
4. Using a compass, use Y as the center and measure of the radius YX, construct an arc intersecting ray BA
5. Using the compass with G as the center and radius still YX, construct an arc intersecting arc DE. Label the point of intersection H
6. Draw \overline{RH}

Justification

When \overline{XY} and \overline{HG} are drawn, $\triangle XBY \cong \triangle HRG$ by SSS, By cpctc,
 $\angle HRG \cong \angle ABC$

Construction 6

Given an angle, construct an angle bisector



Steps

1. Using a compass with B at the center, construct an arc intersecting both rays. Label the intersections as R and S.
2. With points R and S as centers, construct arcs intersecting in the interior of $\angle ABC$. Label the point of intersection as X.
3. Draw ray BX

Justification

When \overline{RX} and \overline{SX} are drawn, $\triangle BRX \cong \triangle BSX$ by SSS, therefore $\angle RBX \cong \angle SBX$ - cpctc and by definition \overline{BX} is an angle bisector.