Dilation
Not an Isometry

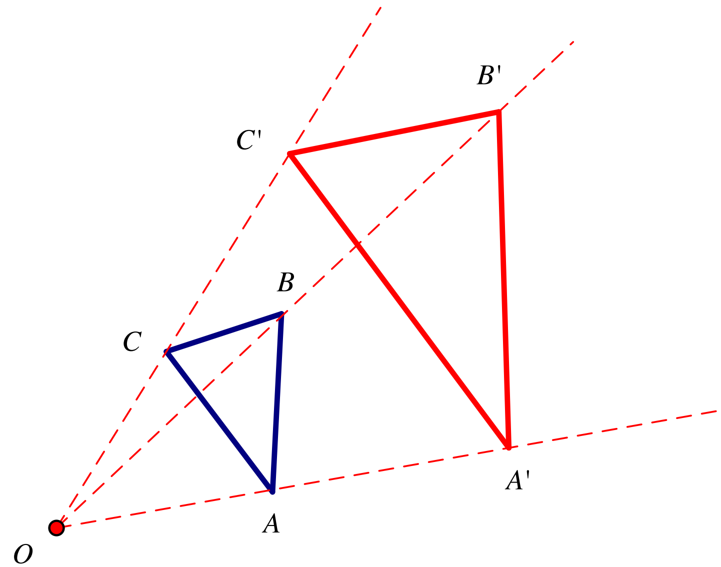
A dilation is a transformation that is not an isometry, it is not a congruence mapping.

A dilation is a transformation that is related to similarity. A good example of a dilation is with the use of a projector. The further the projector is to a wall, the larger the picture. The closer the projector is to the wall, the smaller the picture.

A dilation with center O and scale factor k (k > 0) is a mapping such that:

1. **If P is different form O, then P' lies on $\overrightarrow{OP}$ and OP' = k (OP)**
2. **If P is the point O, then P' is the same point as P**

Beginning with ΔABC and point O, let A' lie on $\overrightarrow{OA}$ so that OA' = 2(OA).
Let B' lie on $\overrightarrow{OB}$ so that OB' = 2(OB), and let C' lie on $\overrightarrow{OC}$ so that OC' = 2(PC)



If k > 1, the dilation is called an **expansion**. If 0 < k < 1, the dilation is a **contraction.**

Writing a dilation mathematically, we have

$$\mathbf{D_{(0,0)\ k}\ (x, y) \longrightarrow (kx, ky).}$$

That is read, a dilation of the point (x, y) with center (0, 0) and scale factor k is mapped into the point (kx, ky)

Example 1 Find the $D_{(0, 0)\ 4}$ (2, 7)

Using the above mapping, we multiply both coordinate by the scale factor 4.
Therefore, we have (8, 28)

Example 2 Find the $D_{(0, 0)\ ½}$ (6, 14)

Using the mapping, we multiply both coordinates by the scale factor.
Therefore, we have (3, 7)

If the dilation is not about the origin, (0, 0), then the mapping is described by:

$$\mathbf{D_{(a,\, b)\, k}\, (x, y) \longrightarrow (a + k(x - a),\, b + k(y - b))}$$

That is read *the dilation of a point (x, y) with center (a, b) and scale factor k is mapped into the point (a + k(x–a), b + k(y–b)).*

Example 3 Find the $D_{(2,3)\,5}$ (4, 6)

Using the above mapping and substituting,
we have (2 + 5(4 – 2), 3 + 5(6 – 3))
(2 + 5(2), 3 + 5(3))
(2 + 10, 3 + 15)
(12, 18)

Example 4 Find the $D_{(1,\,-2)\ \frac{1}{2}}$ (9, 10)

Using the mapping, (1 + ½(9 – 1), –2 + ½(10 – –2))
(1 + ½ (8), –2 + ½(12))
(1 + 4, –2 + 6)
(5, 4)

Example 5 Find the center of the dilation if the preimage was (10, 6), the image was located at (34, 22) and the scale factor was 5.

$(a + k(x - a), b + k(y - b)) \rightarrow a + k(x - a) =$ image x coordinate

$$a + 5(10 - a) = 34$$
$$a + 50 - 5a = 34$$
$$-4a + 50 = 34$$
$$-4a = -16$$
$$a = 4$$

$b + k(y - b) = 22$; y-coordinate

$$b + 5(6 - b) = 22$$
$$b + 30 - 5b = 22$$
$$-4b = -8$$
$$b = 2$$

The center of dilation (a, b) is (4, 2)