EXPONENTIALS

Exponential is a number written with an exponent. The rules for exponents make computing with very large or very small numbers easier. Students will come across exponentials in geometric sequences and other growth models. While probably not aware, the Richter Scale used to measure earthquakes uses exponentials.

In the number 2^4, read two to the fourth power, the four is called the exponent and the two is called the base.

base – 2^4 ← exponent

The ***EXPONENT*** tells you how many times to write the base as a factor.

Examples

1. $2^3 = 2 \times 2 \times 2 = 8$
2. $4^2 = 4 \times 4 = 16$
3. $6^4 = 6 \times 6 \times 6 \times 6 = 1,296$
4. $10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100,000$

Evaluate the following expressions in standard notation.

1. 2^2	2. 3^2	3. 5^3
4. 6^3	5. 10^2	6. 3^5
7. 2^4	8. 5^2	9. 9^2

10. Can you find an easy way of finding the value of an exponential with base 10? (Try a couple and look for a pattern).

So far, so good.

The number 6 could be written as 6^1. What that means is that if a number is written without an exponent, the exponent is understood to be 1.

As an example, 5 is the same as 5^1.

Normally, when we introduce new number sets, like fractions or decimals, etc., we show you how to work with the four basic operations of addition, subtraction, multiplication and division.

Let's start with multiplying exponentials.

Multiplying Exponentials

When we multiply exponentials with the same base, we see a pattern develop. Let's take a look.

***Example* 1** $4^2 \times 4^3$

$4^2 \times 4^3$ means $(4\times 4)\times(4\times 4\times 4)$, writing the answer, we see we are using 4 as a factor 5 times. That is we are multiplying 4 times itself 5 times.

$$\begin{aligned} &4^2 \quad\times\quad 4^3 \\ &(4\times 4) \times (4\times 4\times 4) = 4\times 4\times 4\times 4\times 4 \\ &\qquad\qquad\qquad\qquad = 4^5 \end{aligned}$$

Example 2 $7^5 \times 7^3$

$7^5 \times 7^3$ means $(7\times 7\times 7\times 7\times 7)\times(7\times 7\times 7)$, writing the answer, we see we are using 7 as a factor 8 times.

$$\begin{aligned} &7^5 \qquad\times\qquad 7^3 \\ &(7\times 7\times 7\times 7\times 7)\times(7\times 7\times 7) = 7\times 7\times 7\times 7\times 7\times 7\times 7\times 7 \\ &\qquad\qquad\qquad\qquad\qquad = 7^8 \end{aligned}$$

What if I asked you to simplify the expression $3^{17} \times 3^{8}$, would you want to write all those 3's? No, then let's work smart.

If we were to look at the first two examples, you might see a pattern developing. Can you see it? Let's look

We said $4^2 \times 4^3 = 4^5$. We also said $7^5 \times 7^3 = 7^8$

Do you see anything interesting? Hopefully, you might notice when we multiply exponentials with the **same** base, a way we can find the answer by adding the exponents. Kind of neat, huh?

If that works, then $3^{17} \times 3^{8} = 3^{25}$. Yes, that's a little better than writing out all those three's.

Let's see another, you're saying 5^4 x $5^6 = 5^{4+6}$
$= 5^{10}$

Seeing that does seem to work, we make a rule that will allow us to evaluate those quickly.

Rule 1. ***When you multiply exponentials with the same base, you keep the base and add the exponents.***

Example 3 Simplify in exponential form $8^4 \times 8^5$

$$8^4 \times 8^5 = 8^{4+5}$$
$$= 8^9$$

I know what you are thinking, this is too easy, you are hoping to do dozens of these for homework, right?

Well, try some of them. With a little practice, you can almost simplify all the problems in your head. But, since we want you to remember this rule, do some of them out as we did the last example so you'll be able to recall the rule later.

Simplify In Exponential Notation

1) $5^3 \times 5^4$

2) $6^4 \times 6$

3) $7^5 \times 7^3$

4) $4^3 \times 4^5$

5) $10^8 \times 10^7$

6) $8^3 \times 8^2 \times 8^5$

7) $7^3 \times 7^4$

8) $3^8 \times 3^9$

9) $5^4 \times 5^2 \times 5^7$

10) $2^6 \times 2^5 \times 2^4$

11) $8^2 \times 8 \times 8^4$

12) $3^6 \times 3^{10}$

13) $7^{11} \times 7^4$

14) 8×8

15) $3 \times 3 \times 3 \times 3$

16) 27×3^2

17) $2^3 \times 4 \times 8$

18) 125×5^2

Did you get 8^7 for problem 11? If not, go back and ask yourself how you should get 8^7.

What happens if you don't have the same base? Let's look problems 16 - 18. One way to approach that problem is say it does not fit the rule, the bases are not the same, so I can't simplify it. That sounds okay. But, if I played with this a little longer, I might realize I could write the number 125 as 5^3.

Rewriting the problem, I now have $5^3 \times 5^2$. That's a problem I can do! So, if you are asked to simplify exponentials that don't have the same base, you might try to change the base so you can use the rule.

Dividing Exponentials

If we divide exponentials with the same base, we might see another pattern develop.

Example 1 Simplify in exponential notation $\frac{4^5}{4^3}$

$\frac{4^5}{4^3} = \frac{\cancel{4} \times \cancel{4} \times \cancel{4} \times 4 \times 4}{\cancel{4} \times \cancel{4} \times \cancel{4}}$, dividing out the 4's, we have 4×4 or 4^2

Example 2 Simplify in exponential notation $\frac{7^6}{7^5}$

$\frac{7^6}{7^5} = \frac{\cancel{7} \times \cancel{7} \times \cancel{7} \times \cancel{7} \times \cancel{7} \times 7}{\cancel{7} \times \cancel{7} \times \cancel{7} \times \cancel{7} \times \cancel{7}}$, dividing out the 7's, we have 7 or 7^1

If we were to continue to do some more problems, again a pattern would seem to appear.

Try a couple on your own, do you see a pattern?

What you might be noticing is when you divide exponentials with the same base; we can find the answer quickly by subtracting the exponents. That's good news, in math shortcuts are often turned into things we refer to as rules, formulas, algorithms and theorems.

Rule 2. ***When you divide exponentials with the same base, you keep the base and subtract the exponents*****.**

Example 3 **Simplify** $\frac{9^5}{9^3}$

$$\frac{9^5}{9^3} = 9^{5-3}$$

$$= 9^2$$

Yes, this is really a blast. You're hoping to do more work with exponentials, aren't you?
Well, I've got some good news – we are.

Simplify In Exponential Notation

1) $\frac{4^8}{4^5}$ 2) $\frac{5^3}{5}$ 3) $\frac{10^8}{10^6}$

4) $\frac{3^5}{3^4}$ 5) $\frac{7^3}{7}$ 6) $\frac{9^8}{9^5}$

7) $\frac{12^6}{12^3}$ 8) $8^7 \div 8^3$ 9) $9^8 \div 9^3$

10) $10^7 \div 10^2$ 11) $20^6 \div 20^4$ 12) $15^{12} \div 15^9$

13) $\frac{6^8}{6^5}$ 14) $\frac{3^9}{3^7}$ 15) $3^{12} \div 3^7$

Let's say we want to divide 8^3 by 8^3, what should our answer be?

Zero Exponent

Let's look at that last problem. Using the rule we just developed, we'd have

$$\frac{8^3}{8^3} = 8^{3-3}$$

Oh wow, we have a slight problem. If we use Rule 2, we notice that we get 8 to the zero power; 8^0.

That does not make sense. Remember how we defined an exponent? It tells us how many times to write the base as a factor. How can you write the base zero times? That answer does not seem to make sense, which means the second rule we just developed for dividing does not seem to work.

If we continued to look at samples like that one, for instance;

$$\frac{4^5}{4^5} = 4^{5-5} \text{ or } 4^0$$

We'd also know that anytime we divide a number by itself, we should get one. By the Multiplicative Inverse. $5 \div 5 = 1$;

$12 \div 12 = 1$. That would lead us to believe that $4^5 \div 4^5$ should also be equal to one. As would $8^3 \div 8^3$ be equal to one.

Let's see what we have. We are saying $4^5 \div 4^5 = 4^0$ and we are also saying $4^5 \div 4^5 = 1$. Shouldn't that mean $4^0 = 1$. Looking at $8^3 \div 8^3$ the same way things happens. Using substitution in both of these examples leads us to believe that $4^0 = 1$ and $8^0 = 1$.

Oh yes, you can feel the excitement. We are seeing another pattern develop.

Rule 3. Any number to the zero power, except zero, equals 1.

Why the exception? The rule comes from dividing exponentials and you would never be able to divide by zero in the first place.

Examples

$12^0 = 1$ $\quad\quad$ $(1/2)^0 = 1$ $\quad\quad$ $0^0 \rightarrow$ not defined

Evaluate The Following Expressions

A $\quad\quad$ B $\quad\quad$ C

1) 2^0 12^0 5^0

2) $(1/2)^0$ $(1.54)^0$ 0^0

3) $3^2 - 5^0$ 7^0 128^0

4) $(4+5)^0$ $(6+2)^0$ $(8-2)^0$

5) $(4/3)^0$ $(1/2)^0$ $(6\ 2/3)^0$

We have developed three rules that will allow you to evaluate exponential expressions quickly. Now, instead of writing the rules as we did in word sentences, it might be helpful to learn how to write those same rules using algebraic notation.

For example, in ***Rule 1,*** we said: If you multiply ***exponentials*** with the same base, you add the exponents. Another way to say the same thing is $A^m \times A^n = A^{m+n}$. Notice the base is the same and we added the exponents m and n.

Looking at ***Rule 3,*** rather than writing; Any number to the zero power, except zero, equals one. We could write $A^0 = 1, A \neq 0$ Not letting $A = 0$ takes care of the exception.

Seeing this, how do you think we might rewrite ***Rule 2*** using Algebraic notation?

Rule 2 states: When you divide exponentials with the same base, you subtract the exponents. Remember, the bases have to be the same.

Hopefully you got $A^m \div A^n = A^{m-n}$, $A \neq 0$

Learning how to use mathematical notation is important. It is supposed to make our lives easier. Clearly, using the notation for exponentials will save us some writing time.

IMPORTANT – the rules we learned for multiplying and dividing exponentials only work when you have the **SAME** bases.

In other words $5^3 \times 7^2$ can't be simplified in exponential notation because they don't have the same base and I was not able to make them the same. However, I could evaluate it in standard notation by multiplying 125 x 49.

Rule 1. **$A^m A^n = A^{m+n}$**

Rule 2. **$A^m \div A^n = A^{m-n}$**

Rule 3. **$A^0 = 1$, $A \neq 0$**

Let's combine problems using those rules. We'll see, nothing changes, the rules are the rules. We just have to ensure we have the same bases when simplifying. I would suggest: 1. simplifying the numerators with same bases, 2. Simplifying the denominators, and 3. Use Rule 3, the division rule to simplify the fraction.

Example ***Simplify in exponential notation***

$$\frac{2^3\ 5^2\ 2^4\ 5^7\ 7^8}{2^5\ 5^2\ 7^2\ 5^3\ 7} = \frac{2^7\ 5^9\ 7^8}{2^5\ 5^5\ 7^3}$$

Numerator simplified

Denominator simplified

$$= 2^2\ 5^4\ 7^5$$

Remember, if there is no exponent written, the exponent is understood to b 1.

Evaluate In Exponential Notation

1) $\dfrac{4^2 \bullet 4^5 \bullet 4^3}{4^2 \bullet 4^5}$

2) $\dfrac{7^3 \bullet 7^5}{7^2 \bullet 7^3}$

3) $\dfrac{3^2 \bullet 5^3 \bullet 3^5 \bullet 5^4}{3^4 \bullet 5^2}$

4) $\dfrac{6^7 \bullet 5^8 \bullet 6^4}{6^8 \bullet 5^3}$

5) $\dfrac{5^7 \bullet 10^3 \bullet 5^6 \bullet 10^2}{5 \bullet 10^2 \bullet 5^2}$

6) $\dfrac{6^4 \bullet 7^2 \bullet 7^3}{7^4 \bullet 6^3}$

7) $\dfrac{12^7 \bullet 11^3 \bullet 12^3}{11^2}$

8) $\dfrac{5^2 \bullet 6 \bullet 5^3 \bullet 6^4}{5^5 \bullet 6^3}$

9) $\dfrac{8^2 \bullet 7^3 \bullet 8^4 \bullet 7^2}{8^3 \bullet 7^4}$

10) $\dfrac{8^2 \bullet 3^4 \bullet 8^3}{8^4 \bullet 3^2}$

Negative Exponents

Let's look at another division problem; $\frac{5^2}{5^5}$. Using the last rule since we are dividing exponentials with the same base, we should subtract the exponents.

Example 1 Simplify in exponential notation $\frac{5^2}{5^5}$

$\frac{5^2}{5^5} = 5^{2\text{-}5}$ which equals 5^{-3}

Oh, oh. Having an exponent of -3 does not make sense. Using the definition of exponent, how can we write the base a negative three times? We either write it or don't.

Since we are running into difficulty, let's do the problem the long way (by definition) and see what's happening.

$\frac{5^2}{5^5} = \frac{\cancel{5} \times \cancel{5}}{\cancel{5} x \cancel{5} \times 5 \times 5 \times 5}$, dividing out the 5's, we have $\frac{1}{5 \times 5 \times 5} = \frac{1}{5^3}$

Another example might be in order.

Example 2 Simplify in exponential notation $\frac{7^4}{7^6}$

$\frac{7^4}{7^6} = 7^{4\text{-}6}$ which equals 7^{-2}. Doing it by the definition of exponent, we get,

$\frac{7^4}{7^6} = \frac{\cancel{7} \times \cancel{7} \times \cancel{7} \times \cancel{7}}{\cancel{7} \times \cancel{7} \times \cancel{7} \times \cancel{7} \times 7 \times 7}$, dividing out the 7's, we have $\frac{1}{7 \times 7} = \frac{1}{7^2}$

Looking at those answers, we see that 5^{-3} and $\frac{1}{5^3}$ are the answers to the first problem.

By the same token, we see 7^{-2} and $\frac{1}{7^2}$ are answers to the second example. Uh-huh, another pattern you're thinking. You've got it, another pattern, another rule.

Rule 4. **Any base, except 0, raised to a negative exponent is equal to 1 over the base raised to a positive exponent Or $A^{-n} = \frac{1}{A^n}$, $A \neq 0$**

Notice a negative exponent does not mean the number is negative.

Write the answer using a negative exponent

1) $\frac{5^2}{5^6}$	2) $\frac{3^4}{3^9}$	3) $\frac{7^3}{7^5}$
4) $\frac{10^3}{10^7}$	5) $\frac{8^4}{8^5}$	6) $\frac{7^3}{7^{10}}$
7) $\frac{2^5}{2^8}$	8) $\frac{4}{4^3}$	9) $\frac{x^4}{x^7}$

Write the answer in fractional form using a positive exponent.

10) 4^{-2} 11) 5^{-3} 12) 10^{-1}

13) 10^{-2} 14) 3^{-4} 15) 7^{-5}

16) 10^{-3} 17) 2^{-3} 18) 6^{-5}

Raising a Power to a Power

Let's continue playing. I bet we can find more patterns that will allow us to simplify exponential expressions quickly.

Example 1 Simplify in exponential notation $\left(5^2\right)^3$

The exponent 3 tells me how many times to write 5^2 as a factor.

$\left(5^2\right)^3 = 5^2 \times 5^2 \times 5^2$

Using Rule 1, when we multiply exponentials with the same base, we add the exponents,

$$5^2 \text{ x } 5^2 \text{ x } 5^2 = 5^6$$

we find that $\left(5^2\right)^3 = 5^6$

Piece of cake, don't you think? Let's look at another one.

Example 2 Simplify in exponential notation $\left(4^3\right)^5$

$$(4^3)^5 = 4^3 \times 4^3 \times 4^3 \times 4^3 \times 4^3$$
$$= 4^{15}$$

Looking at those two examples, how do you think $(2^{25})^4$ can be written? If you look at the problem and the answer, the pattern almost jumps out at you. If you are thinking 2^{100} then we have identified another pattern. Say Yes! to mathematics, this is really fun.

When you raise a power to a power, the pattern suggests we multiply the exponents. That leads us to another shortcut, another rule.

Rule 5. **When a power is raised to a power, keep the base and multiply the exponents.** $(A^n)^m = A^{nm}$

Example 3 Simplify in exponential notation. $(6^4)^3$

Using **Rule 5** $(6^4)^3 = 6^{4x3} = 6^{12}$

Simplify the Following Expression In Exponential Notation

1)	$(5^2)^3$	2)	$(4^3)^5$
3)	$(7^4)^2$	4)	$(10^2)^4$
5)	$(8^5)^2$	6)	$(2^{10})^8$
7)	$(3^4)^5$	8)	$(3^5)^4$
9)	$(3^{-2})^5$	10)	$(5^3)^{-4}$

How is $(2^3\ 5^2)^4$ the same as the ones we just completed? How is it different? Using the definition of exponent, using what is in parentheses (base) as a factor four times, we have

$(2^3\ 5^2)\ (2^3\ 5^2)\ (2^3\ 5^2)\ (2^3\ 5^2)$ which is $2^{12}\ 5^8$ using the rule for multiplying. Does this fit as a slight variation to the rule for raising a power to a power?

We have established 5 rules by identifying patterns. At first glance, if you didn't see the pattern develop, the rules (shortcuts) would not make sense. But since we saw those patterns develop, we are feeling pretty good about them. Aren't we?

Now because we also saw how much easier it was to write the rules using mathematical notation than in long hand, we are really starting to get into this.

Rule 1. $A^m \times A^n = A^{m+n}$

Rule 2. $A^m \div A^n = A^{m-n}$, $A \neq 0$

Rule 3. $A^0 = 1, A \neq 0$

Rule 4. $A^{-n} = \frac{1}{A^n}$, $A \neq 0$

Rule 5. $\left(A^n\right)^m = A^{nm}$

Next, let's see what happens when we have longer problems that require us to use more than one rule per problem as we did with the first three rules. Excited?

Let's use a few of these rules in one problem.

Example 4 Simplify in exponential notation $\frac{3^4 \times 4^3 \times 5^2 \times 3^6 \times 5^3}{3^7 \times 4^5 \times 5^5}$

Simplifying the numerator by using Rule 1- same bases, we get $3^{10} \times 4^3 \times 5^5$

$$\frac{3^4 \times 4^3 \times 5^2 \times 3^6 \times 5^3}{3^7 \times 4^5 \times 5^5} = \frac{3^{10} \times 4^3 \times 5^5}{3^7 \times 4^5 \times 5^5}$$ Continuing using Rule 2

$$= 3^3 \times 4^{-2} \times 5^0$$
$$= 3^3 \times \frac{1}{4^2} \times 1$$
$$= \frac{3^3}{4^2}$$

When you simplify exponentials, you cannot have a zero or negative exponent.

A Change in Base is Possible

The rules we developed work with exponentials having the SAME base. At this point in math, if the bases are not the same, we cannot simplify them using the rules. However, there are times you can make the bases the same. For instance, at first glance, we might think $25 \text{ x } 5^4$ cannot be simplified because the bases are not the same. That's almost true. If we think about it, we might realize we can change 25 to 5^2, thus changing the problem to $5^2 \text{ x } 5^4$, exponentials with the same base; so that problem could be simplified.

Example $25 \text{ x } 5^4$

$5^2 \text{ x } 5^4 = 56$

Example $9^3 \text{ x } 27^4$

Since they don't have the same base, it appears we can't use our rule for multiplying, but … we see that can be written as $9 = 3^2$ and $27 = 3^3$., we can substitute those into our problem.

$(3^2)^3 \text{ x } (3^3)^4$

$= 3^6 \text{ x } 3^{12}$

$= 3^{18}$

Rewriting Radicals as Exponentials

We have rules for operating with radicals, it can sometimes become problematic. For example, simplifying $\sqrt[2]{x} \cdot \sqrt[3]{x}$ looks like its simplified because they don't have the same indices.

Looking at a more familiar problem that can be simplified

Example $\sqrt[2]{x^1} \cdot \sqrt[2]{x^1} = \sqrt[2]{x^2} = \text{x}$

Playing with more examples like that, we might see a pattern that would allow us to write radicals as exponentials – excited? Using the index as the denominator and exponent in the radicand as the numerator, we have

$\sqrt[2]{x^1} \cdot \sqrt[2]{x^1} = \sqrt[2]{x^2} = x$

$x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} = x^1$ That interesting and leads us to be able to rewrite radical in exponential notation.

$$\sqrt[n]{x^p} = x^{\frac{p}{n}}$$

When the index is not written it is understood to be 2.

$$\sqrt[2]{x} = \sqrt{x}$$

Remember, the exponent is the numerator, the index is the denominator.

Rewrite the following as radicals or as exponentials with rational exponents**.**

1. $4^{1/2}$
2. $8^{2/3}$
3. $X^{2/5}$
4. $(3Y)^{1/3}$
5. $\sqrt[2]{16^1}$
6. $\sqrt[3]{8^2}$
7. $\sqrt[5]{X^2}$
8. $\sqrt[4]{X^2}$
9. $\sqrt[4]{X^2Y^8}$
10. $\sqrt[3]{27X^2Y^9Z^3}$

Exponential Equations

This next theorem, with a little thought, would seem to make sense. I the bases are equal, then the exponents would have to be equal.

Theorem **For b > 0, b ≠ 1, $b^x = b^y$ if and only if x = y**

Example **$3^x = 3^{4,}$ so x = 4**

It would appear, using that theorem, that if we had exponential equations and could set the bases equal, we could then set the exponents equal and solve the equation.

Let's write that as an algorithm

Algorithm for Solving Exponential Equations

1. Express each side of the equation as a power in the SAME base.
2. Simplify the exponents
3. Set the exponents equal
4. Solve the resulting equation

Example 1 Find the value of x; $2^5 = 2^{2x-1}$

The bases are already equal, so set the exponents equal.

$$5 = 2x + 1$$
$$4 = 2x$$
$$2 = x$$

Example 2 Find the value of x; $5^x = 125$

The bases are not equal, so rewrite 125 as 5^3, then solve

$$5^x = 5^3$$
$$x = 3$$

As is always the case in math, I can't make these problems harder – only longer.

Example 3 **Solve:** $2^{6x^2} = 4^{5x+2}$

Using the algorithm, I will make the bases equal

$$2^{6x^2} = (2^2)^{5x+2}$$

$$2^{6x^2} = 2^{10x+4}$$

Now that the bases are the same, we set the exponents equal/

$$6x^2 = 10x + 4$$

Simplifying that, we have $3x^2 - 5x - 2 = 0$

Factoring $(3x + 1)(x - 2) = 0$

Setting equal to zero, so $x = -\frac{1}{3}$ and $x = 2$

Try a few

1. Solve for x. $9^{3x} = 27^{x-2}$

2. Solve for n. $9^{n-1} = (1/3)^{4n-1}$

3. Solve: $4(2^x) - 6 = 58$

Okay, we'll let you try some of these combined problems on your own. We won't make you beg for more.

WRITE IN STANDARD NOTATION

1) $2^3 =$

2) $4^2 =$

3) $1^8 =$

4) $8^0 =$

5) $7^0 \div 7^1 =$

6) $13^1 \times 52^0 =$

WRITE IN EXPONENTIAL NOTATION

7) $5^2 \bullet 5^4 =$

8) $6^3 \bullet 6^7 =$

9) $2^5 \bullet 2^7 =$

10) $\frac{8^4}{8^3} =$

11) $10^7 \div 10^5 =$

12) $\frac{7^3}{7} =$

13) $\frac{3^4 \bullet 3^6}{3^8} =$

14) $\frac{4^7 \bullet 4^5}{4^3} =$

15) $\frac{8^4 \bullet 8^5}{8^6 \bullet 8^2} =$

16) $3^2 \bullet 4^5 \bullet 3^7 =$

17) $8^2 \bullet 4^3 \bullet 8^4 \bullet 4^2 =$

18) $\frac{6^7 \bullet 8^2 \bullet 6^4}{8 \bullet 6^8} =$

19) $3 \bullet 3 \bullet 3 \bullet 3 \bullet 3 =$

20) Redo #17, using base 2

Evaluate In Exponential Notation

1)

$$\frac{4^2 \bullet 4^5 \bullet 4^3}{4^2 \bullet 4^5}$$

2)

$$\frac{7^3 \bullet 7^5}{7^2 \bullet 7^3}$$

3)

$$\frac{3^2 \bullet 5^3 \bullet 3^5 \bullet 5^4}{3^4 \bullet 5^2}$$

4)

$$\frac{6^7 \bullet 5^8 \bullet 6^4}{6^8 \bullet 5^3}$$

5)

$$\frac{5^7 \bullet 10^3 \bullet 5^6 \bullet 10^2}{5 \bullet 10^2 \bullet 5^2}$$

6)

$$\frac{6^4 \bullet 7^2 \bullet 7^3}{7^4 \bullet 6^3}$$

7)

$$\frac{12^7 \bullet 11^3 \bullet 12^3}{11^2}$$

8)

$$\frac{5^2 \bullet 6 \bullet 5^3 \bullet 6^4}{5^5 \bullet 6^3}$$

9)

$$\frac{8^2 \bullet 7^3 \bullet 8^4 \bullet 7^2}{8^3 \bullet 7^4}$$

10)

$$\frac{8^2 \bullet 3^4 \bullet 8^3}{8^4 \bullet 3^2}$$

Simplify

1)

$$\frac{5^2 \bullet 3^4 \bullet 5^6 \bullet 3^8}{5^6 \bullet 3^{10}}$$

2)

$$\frac{4^5 \bullet 8^6 \bullet 4^3 \bullet 8^3}{4^5 \bullet 8^4}$$

3)

$$\frac{4^5 \bullet 10^6 \bullet 4^2 \bullet 10}{4^5 \bullet 10^4}$$

4)

$$\frac{5^3 \bullet 5^4}{5^1 \bullet 5^2}$$

5)

$$\frac{4^2 \bullet 5^3 \bullet 4^2 \bullet 5^5}{4^3 \bullet 5^7}$$

6)

$$\frac{9^2 \bullet 9^8 \bullet 10^7}{10^6 \bullet 9^4}$$

7)

$$\frac{6^2 \bullet 5^3 \bullet 6^4 \bullet 5^4}{5^2 \bullet 6^5 \bullet 5}$$

8)

$$\frac{2^4 \bullet 8^6 \bullet 2^5 \bullet 8^4}{8^9}$$

9)

$$\frac{10^7 \bullet 2^3 \bullet 10^8}{10^2 \bullet 2 \bullet 10^3}$$

10)

$$\frac{5^2 \bullet 4^8 \bullet 5^3 \bullet 4}{5^4 \bullet 4^6}$$

Evaluate In Exponential Notation

1) $9^3 \bullet 9^2$

2) $8^4 \bullet 8^5$

3) $7^5 \times 7^6$

4) $\frac{9^8}{9^6}$

5) $\frac{10^7}{10^4}$

6) $3^4 \div 3^3$

7) $\frac{8^2 \bullet 8^4}{8^3}$

8) $\frac{10^6}{10^2 \bullet 10^3}$

9) $\frac{8^2 \bullet 8^4}{8^6}$

10) $10^6 \div 10^4$

(11) $8^3 \bullet 8^5$

12) $\frac{13^8}{13^2}$

13) $\frac{7^6 \bullet 2^5}{2^8}$

14) $\frac{8^2 \bullet 2^6 \bullet 8^3 \bullet 2^4}{8^4 \bullet 2^{10}}$

15) $\frac{10^2 \bullet 3^4}{10 \bullet 3}$

16) $\frac{4^2 \bullet 5^2}{4 \bullet 5}$

17) $\left(3^2\right)^3$

18) $\left(5^6 4^3\right)^2$

Exponentials Name
Date

Definitions

1. ***Exponential

2. ***Exponent

3. ***In the number 6^3, the 3 is called the _______________ and the 6 is called the _______________ .

4. ***Write the five rules for working with exponentials.

5. ***Write the procedure for solving exponential equations

Evaluate in standard form.

6.** 5^2

7.** 3^4

8.** -2^2

9.** $(2/5)^{-1}$

Simplify in exponential notation.

10.** $5^3 \times 5^4$

11.** $6^5 \div 6^3$

12. 7^0

13.** $(5^4)^2$

14.** $5^3 \div 5^7$

Simplify in exponential notation.

15.** $5^4 \times 6^3 \times 5^2 \times 6^4$

16.** $\dfrac{5^5 \times 6^3 \times 5^2 \times 6^7}{5^4 \times 6^6}$

17.** $\frac{2^3 \times 4^3 \times 2 \times 4^5}{2^2 \times 4^8}$

18.** Which fraction is larger?

a. $(1/2)^3$ b. $(1/2)^4$

19.* Show that $(2/3)^{-5} = (3/2)^5$
Hint: use the rule for negative exponents and simplify.

20.* Use an example to show that $a^m \div a^n = a^{m-n}$

21.* Simplify in exponential notation. $25^2 \times 125$

22.* Why is any number to the zero power, except zero, equal to one?

23.** Solve: $2^{3x+1} = 2^{x-5}$

24.** Solve $9^{3x} = 27^{x+1}$

25.*** Provide a parent/guardian contact information; phone, email, etc (CHP)