SETS & Venn Diagrams

The idea of sets shouldn't be anything new to you. You have probably been classifying things all of your life. We might talk about all redheads with blue eyes or list all the states that begin with the letter "A" in the United States. Those are both examples of sets. Another example of a set is the people going to a particular school

Mathematically, we define a set to be a **well-defined collection objects**. The objects are often referred to as members or elements of the set.

Braces are used to enclose the elements of a set and we label the set with a capital letter. The set of single digit numbers can be written as:

$$A=\{0,1,2,3,4,5,6,7,8,9\}$$

The order in which the elements are listed does not matter.

We use the symbol $\in$ to show an element belongs to a set or is a member of that set. For example $2 \in A$. To show an element does not belong to a set, we use the same symbol with a line through it; $\not\in$. Piece of cake, don't you think?

Sometimes making a complete listing of the elements of a set might not be warranted because of the sheer number of elements. In that case, we develop more notation. In those cases we use something called **set builder notation**. Using set builder notation to describe single digit numbers would look like this:

$$A=\{x \,/\, x \text{ is single digit number}\}$$

The way you say that is "A" is the set of all elements *x* such that *x* is a single digit number. The first "*x*" inside the brackets just identifies the variable be used to describe the elements. The "/" is read "such that". Try saying this one.

$$B=\{x \,/\, x \text{ is a prime number less than ten}\}$$

The way you'd say that is "B is the set of all *x's* **such that** *x* is a prime number less than ten." Or I could have chosen to list them.

$$B=\{2,3,5,7\}$$

As you know, one is not a prime number

A set that has no elements is called the empty set or null set. We use the symbol ∅ or brackets with nothing in them { } to indicate that.

An example of an empty set would be all the states that begin with the letter z.

The universal set is the set that contains all the elements being considered in a discussion. The universal set is denoted by **U**. It's important that you recognize the universal set in any given problem.

Let's say we are discussing the letters a, b, and c. The universal set could be all the letters in the alphabet So

$$U = \left\{ x \,/\, \text{is a letter in the alphabet} \right\}$$

Now, if the only letters I am actually discussing are a, b, and c, then I could describe those letters in set notation.

$$T = \left\{ a, b, c \right\}$$

I picked "T" because I didn't want coffee. That's a joke. Ok, what happens if I want to discuss the letters that are not elements of T? Well, you know that would be the letters d through z. When you want to talk about members or elements not in a particular set, we call that the complement of the set. There are a number of ways to show this symbolically; $\sim T, T'$, or T.

I'm going to use the $\sim T$ notation. Now back to set T, {a,b,c}. If we want to talk about the letters not in T, we write

$$\sim T = \left\{ x \,/\, x \in U \text{ and } x \notin T \right\}$$

So the **complement** is made up of all the members in the universal set that are not members of T.

I know, you are thinking learning all this notation is pretty cool, so we'll show you some more.

Let's say we are talking about the numbers one through ten. We could call that our universal set $U = \left\{ 1,2,3,4,5,6,7,8,9,10 \right\}$ and let's say we then look at just the odd numbers less than ten, call it **A**.

$A = \{1,3,5,7,9\}$. We might also want to discuss another set, the prime numbers greater than two, but less than ten, call that **B**. $B = \{3,5,7\}$. Notice that all the elements of B are contained in A. Then B is called a subset of A, written $B \subseteq A$.

So now we have another definition, a subset, **B is a subset of A if and only if all the elements in B are also elements of A.**

If B is a subset of A and B is not equal to A, then B is a proper subset of A, written $B \subset A$. In other words, B would have to have fewer elements.

Let's say I had another set $G = \{0,5,7\}$, would G be considered a subset of A? Are all the members of G also members of A? We can see that "0" is not in A, therefore it would not be a subset. So we write $G \not\subset A$.

I need you to stay with me, subsets and elements of a set can be confusing for the novice. If we want to talk membership, or an element belonging to a set, we use $\in$.

So we could say that $3 \in A$, 3 is a member of A. Since three is not a set, we can **NOT** substitute $\subset$ for $\in$. However, if we are talking about 3 as a set, we could write $\{3\} \subseteq A$.

In other words, if we are talking sets we have to use either capital letters or $\subset$. If we are just talking membership, we use $\in$ and don't put brackets around the element. For example, $3 \in A$.

Now, the question is, what differentiates the two, membership versus being a set? Membership means you just belong, to be considered a set, it's more than just belonging, if something is to be considered a set, it must be well defined, it must be distinguishable.

That's important, so read it again.

Now that we have the basic definitions and notation out of the way, we can play.

Set Intersection

We'll start with set intersection. There's no need to get worried, you have probably used the concept of set intersection many times in your life, we're just going to describe it mathematically now. Excited?

Let's say a college representative wants to send information to all students that are enrolled in calculus and physics. To do this, they would have to identify students taking both classes. That group of students would be identified as the intersection of the two sets – kids taking calculus, C and the other set, kids taking physics, P.

Mathematically, we'd write the intersection of two sets C and P, written $C \cap P$, is the set of all members common to both C and P.

$$C \cap P = \{x / x \in C \text{ and } x \in P\}$$

Students not enrolled in both classes would not be in the intersection.

The key in the definition of intersection is the word **and.** In everyday language, as it is in math, and means both conditions must be met.

If sets such as C and P have no members in common, they are called **disjoint** sets. In other words, their intersection is the null set, $C \cap P = \varnothing$

Let's look at an example.

EXAMPLE 1 Let U= {letters in the alphabet}

M = { q, r, s, t, z}, N = { s, p, o, t}

Find M ⋂N.

What members are common to both sets? Or another way of asking, which letters are in both set M and set N? The intersection is {s,t}, so M ⋂N.= {s, t}

. **Example 2** Find $A \cap B$, if $A = \{d, e, f, g\}$ and $B = \{d, o, g\}$

Since we want to find the intersection, we are looking for elements common to both. So looking at the sets, which elements, if any, belong to both sets A and B?

Hopefully, you noticed *d* and *g* are in both sets, therefore

$$A \cap B = \{d, g\}$$

Set Union

Other things that happen in our life might be described as the set union. Going back to the two sets, C is the set of kids taking calculus, P is the set taking physics. If we would like to contact students taking calculus or physics, then we would be talking about the set union. That's the kids taking calculus or physics or both calculus and physics..

$$C \cup P = \{x \,/\, x \in C \text{ or } x \in P\}$$

The key word in the definition is *or*. In math, *or* means "one or the other or both". That's called the inclusive *or*. Many of us use the exclusive *or* at home, which means to say one or the other but not both. So *or* in math is used differently than the *or* at home.

Let's look at a couple of examples.

That's just too easy! Let's look at the union of the sets A and B.

Example 3 Find $A \cup B$, if $A = \{d, e, f, g\}$ and $B = \{d, o, g\}$

To find the union we are looking for elements that belong to either set. That would include elements that belong to both sets. What that means is we join the two sets together, therefore

$$A \cup B = \{d, e, f, g, o\}$$

In the set union, any element that belongs to either set gets to join the set union. But in this club, these members (elements) only get one vote. So notice, I did not repeat the elements *d* and *g* in the set union.

Set Difference

Another operation with sets is called the set difference. So let's go back to the calculus and physics students we described earlier by the sets C and P. If we wanted to talk about the set of students taking calculus but not physics, we'd write **C – P**.

Let's see what that would look like if we described these members in set notation.

If $C = \{a, b, c, d, e\}$ and $P = \{b, d, f, g, h\}$, then we'd only want the people taking calculus that are not enrolled in physics.

Let's look at one student at a time, *a* is in calculus, but not physics. So *a* works out.

b is in calculus, but he's in physics also, so *b* won't belong.

c is in calculus, but not in physics, so *c* is OK. How about *d*? *d* is in calculus, but he's also in physics, so *d* won't belong.

e is in calculus, he's not in physics, so e is OK to join the set difference.

Another way to look at that set difference to write the members of the calculus class, and tell anyone in that class if they are also taking physics, they have to leave

C—P = {a,c,e)

I know what you are thinking, you want to try some of these on your own.

If $U = \{a,b,c,d,e,f,g\}$, $A = \{d,e,f\}$, and $B = \{a,b,c,d,e\}$, find each of the following.

1. $A \cup B$
2. $A \cap B$
3. $A - B$
4. $B - A$
5. $\sim A$
6. $.\sim (A \cap B)$

That was fun! Let's see how you did.

$A \cup B = \{a,b,c,d,e,f\}$. The union joins the two sets together.

$A \cap B = \{d,e\}$. The intersection uses only members that belong to both sets.

$A - B = \{f\}$. The set difference takes all the members of A sends members that belong to B home.

$B - A = \{a,b,c\}$. The set difference takes all the members of B and sends members that belong to A home. Just like a subtraction problem, that's probably why they call it the set difference.

$\sim A = \{a,b,c,g\}$ The complement of A means all the members of the universal set except members that belong to A.

$\sim(A \cap B) = \{a,b,c,f,g\}$. The complement of the intersection of A and B means everyone in the universal set, except those that belong to the intersection. We already found $A \cap B = \{d,e\}$, the complement is the set that contains all the members of the universal set, except members *d* and *e*.

Those were just too easy. Would $A \cup B = B \cup A$? How about $A \cap B = B \cap A$?

It turns out the answer to those questions is yes. So it appears we have a commutativity property for set union and set intersection.

Does $A - B = B - A$? Going back to problems we just did and looking at the answers, we can see that is not true. The set difference is not commutative.

Can I make these problems more difficult? Absolutely not, I can make them longer by having more sets, but I can't make them harder.

Let me prove that, I'll make a problem longer – but as you will see, not more difficult!

Example 4 Find $A \cup B \cap C$

Let U = $\{m,a,v,e,r,i,c,k\}$, $A = \{r,i,c,k\}$ $B = \{v,e,r,i\}$ and we'll make a third set C. $C = \{m,i,r,e\}$

Well, just like in arithmetic, when we don't have parentheses we work from left to right. So, let's find $A \cup B$.

$$A \cup B = \{r,i,c,k,v,e\}$$

Now let's intersect that with C.

$\{r,i,c,k,v,e\} \cap \{m,i,r,e\} = \{r,i,e\}$, therefore

$$A \cup B \cap C = \{r,i,e\}$$

That could have been written with parenthesis; $(A \cup B) \cap C$

Let's change that problem by moving the parenthesis and see if the answer is different..

Example 5 Find $A \cup (B \cap C)$.

Just like in arithmetic, we perform the operation in parenthesis first.

$$B \cap C = \{i, r, e\}$$

Now let's join that with A.

$$\{r, i, c, k\} \cup \{i, r, e\} = \{r, i, c, k, e\}, \text{ therefore}$$
$$A \cup (B \cap C) = \{r, i, c, k, e\}$$

That wasn't hard. All we had to do is perform the first operation, then perform the second operation using that answer.

Now, let's look at the same types of problems, with the same definitions, and operations using visuals – Venn Diagrams

VENN DIAGRAMS

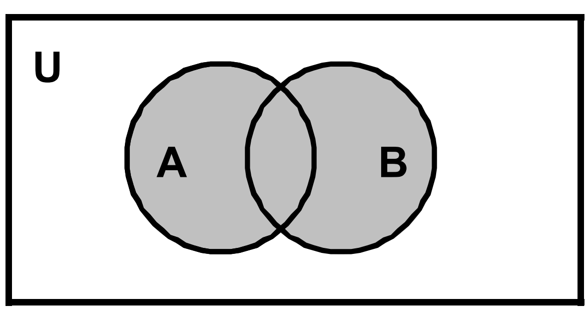
In the last section we represented sets using set notation, listing the elements in brackets. With Venn Diagrams, we define everything exactly the same way, but the definitions are in the form of pictures.

For instance, the set union of A and B, written $A \cup B$ was defined as:

$$A \cup B = \{x \,/\, x \in A \text{ or } x \in B\}$$

With Venn Diagrams, the definition again is all the members that belong to A or B, but we show that by shading in the circles A and B.

A U B



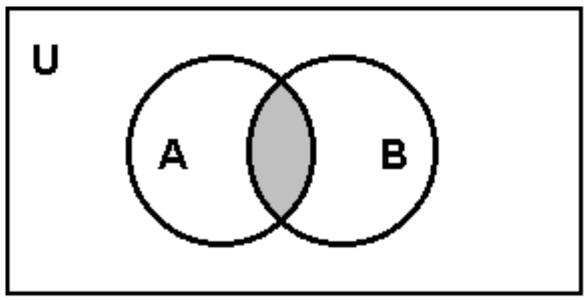
That's important, the set union is defined by shading in both circles of the Venn Diagram, all the members belong.

Let's look at the set intersection of A and B, that was defined as the members that belonged to both sets.

$$A \cap B = \{x \,/\, x \in A \text{ and } x \in B\}$$

That is illustrated by shading only the portion of the circles that overlap. Those elements belong to A and also to B.

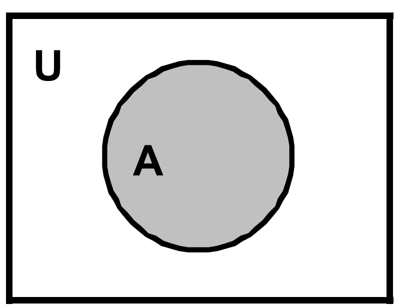
A ∩ B



Notice, there were rectangles around those examples. Those rectangles represented the universal set. All the elements under discussion.

To represent a single set, such as A. I would draw one circle and shade it in.

A

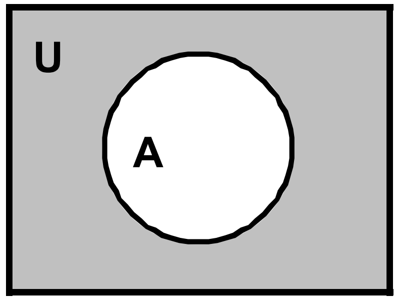


How would the complement of A, ~A be illustrated? I'm glad you asked. Just like before, the complement of A are all the members of the universal set not in A.

$$\sim A = \{x \,/\, x \in \bigcup \text{ and } x \not\in A\}$$

That's illustrated by shading in the rectangle, but not any part of the circle.

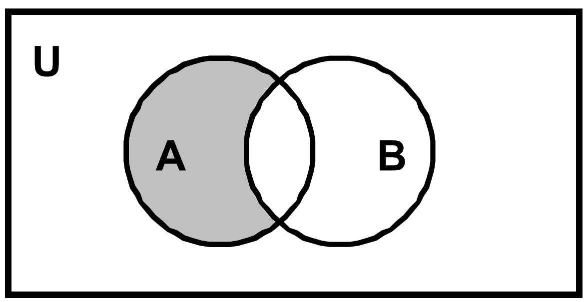
~**A**



I know, you love coloring, you want to do more. Let's look at the set difference. Remember A – B was defined as all the elements in A, but couldn't belong to B.

$$A - B = \{x / \in A \text{ and } x \notin B\}$$

A – B



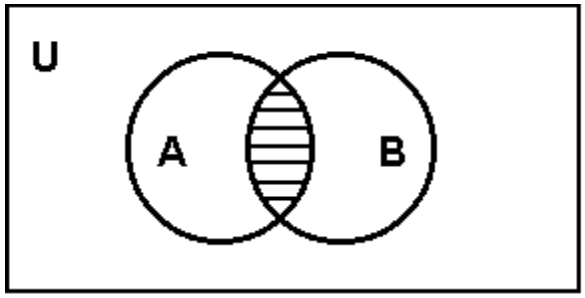
Having those operations defined through Venn Diagrams, we can now play with more sets. Again, the illustrations are important, introducing a third set using a circle does not in any way change what we have already defined. In fact, if there is a third set, we work with just two at a time, just like we did with sets.

So, let's scaffold by adding a 3rd set – a 3rd circle. We can make problems longer – but not harder.

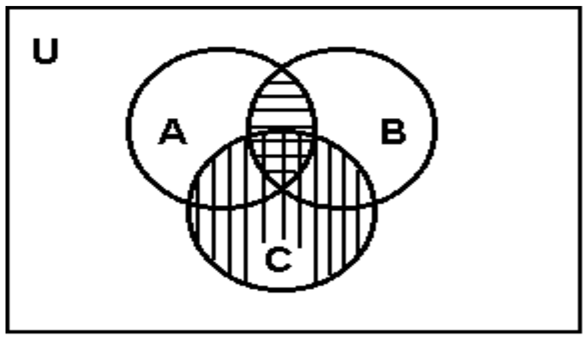
We will do these like we do all math, using the Order of Operations, etc.

Example Shade $A \cap B \cap C$

First we'll find the intersection of A and B, completely ignoring C. After that, we take that result and intersect it with C. We already know be definition, that $A \cap B$ looks like this

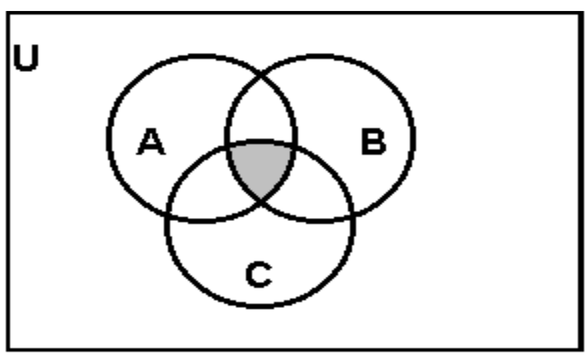


Now, we'll take that shading and intersect that with circle C.



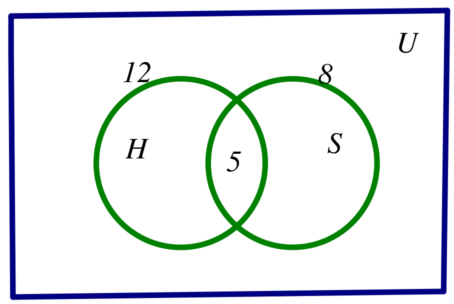
I shaded $A \cap B$ with horizontal lines and C with vertical so you can tell the difference. Where the shading overlaps is what these sets have in common. That almost makes sense, for an element to belong to A, B and C, there is only one region within the three circles that satisfies that.

$$A \cap B \cap C$$



Example 1 There are 19 boys who belong to the Breakfast Club. 12 like ham, 8 like sausage and 5 like both ham and sausage. How many in the club like ham only? Only like sausage?

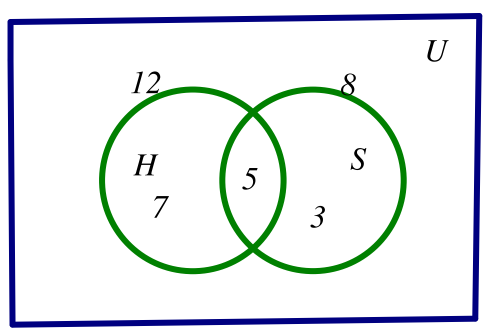
The best way to start these types of problems is to create a Venn Diagram labeling the circles as H for ham and S for sausage. The intersection of the circles represents the members of the club that like both ham and sausage.



Now if there are 12 club members who like ham and 5 are already accounted for because they like ham and sausage, how many like only ham? 12 – 5 = 7

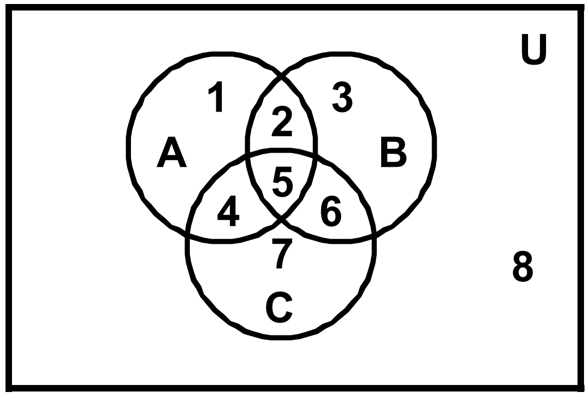
Using the same argument, how many like just sausage? 8 – 5 = 3

Let's fill in the Venn Diagram.



When you add the numbers of who likes ham only, sausage only and ham and sausage, that totals 15. There are 19 members in the club, where are the other 4?? They would be located outside the circle because they don't like ham or sausage.

Let's look at a Venn Diagram made up of three sets in which the regions are labeled. Now, we'll describe each region.



Region 5 is in all three circles. So any elements in region 5 would belong to all three sets. In other words; $A \cap B \cap C$.

What about Region 2? Those are the elements in A and B, but not C. How might you describe Region 6? Those are elements in B and C, but not in A. Try Region 4. The elements in A and C, but not B.

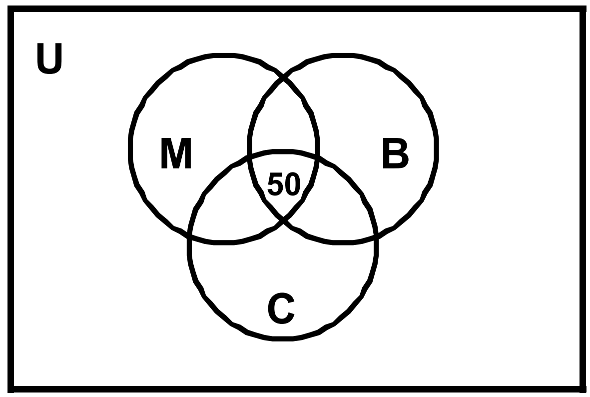
This is fun, let's look at some more regions. Region 1 describes the elements in A only. What about region 3? Those elements are only in B. Region 7 then would be the elements in C only. Region 8 would describe elements that are not members of any of the sets, but belong to the universal set.

It's important that you become familiar with how each of those regions might be described. Being able to describe those regions would allow you to solve some problems.

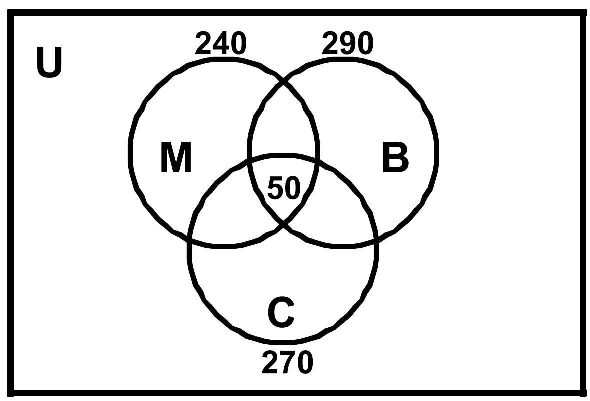
Example 2 A survey was taken of 650 university students. It was reported that 240 were taking math, 290 were taking biology, and 270 were enrolled in chemistry. Of those students, 80 were taking biology and math, 70 were taking math and chemistry, 60 were taking biology and chemistry, and 50 were taking all three classes. How many students took math only?

At first glance, you might not think this is possible because the numbers add up to more than 650. But if you are familiar with how the regions are described, we can determine how many were in each region.

In going about this problem, I would tell you to draw a Venn Diagram and begin by filling in Region 5, the students that took all three courses.



After doing that, we'll place the number of students taking each course on the circle because we don't know where those students should be located within the circles.



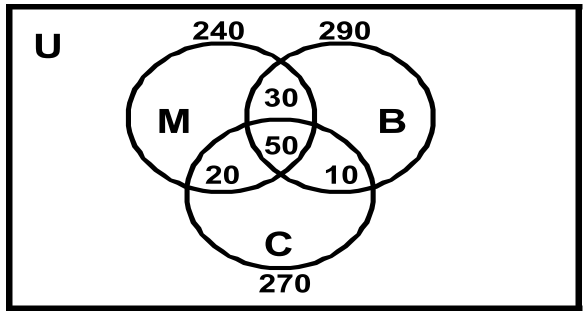
Okay, now we can have some fun by determining what regions the students should be located.

For instance, it says that 80 students are taking math and biology. We have 50 of those accounted for in Region 5, how many does that leave to be in Region 2? $80 - 50 = 30$

That's easy enough. Using that same reasoning, 70 students are taking math and chemistry, how many students would then be in Region 4? Well, $70 - 50 = 20$. That's pretty easy, don't you think?

Ok, how many students should be in Region 6? Since there are 60 students enrolled in biology and chemistry, and 50 of them are accounted for in Region 5, that leaves 10 students for Region 6.

Let's fill in those numbers:

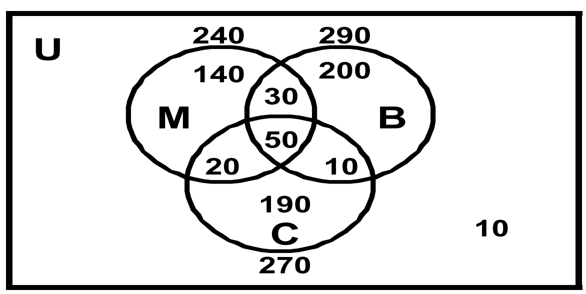


Now how many students would be in Region 1? Now remember, there are supposed to be 240 students taking math, we have 100 accounted for in Regions 2, 4, and 5. That leaves 140 students in region 1, taking math only.

How many students are taking biology only? Well, we were told that 290 students were taking biology, we have 90 of them accounted for in Regions 2, 5, and 6, that leaves 200 students in Region 3.

How many are taking only chemistry? We know there are 270 students taking chemistry, we have 90 accounted for in Regions 4, 5, and 6, that leaves 190 in Region 7.

Filling in those numbers and taking the numbers off the circle, we have the following information.



We have one slight problem, if we add those regions within the circles, the total is 640 students. The problem stated 650 were surveyed, we're missing ten students. Where are they? That's right, they would be in Region 8, not taking any of those courses.

Now tell me, was that fun?

How many students took math and biology, but not chemistry?

How many students took math and chemistry, but not biology?

How many students took biology and chemistry, but not math?

How many students took only math?

How many students took exactly two of the courses?

Now, let's make a point. Solving these problems require you to understand the language and translate that to math and to be able to add and subtract. My point is the math is not hard.

VENN DIAGRAMS

1a. A survey of 70 high school students revealed that 35 like folk music, 15 like classical music, and 5 like both. How many of the students surveyed do not like either folk or classical music?

2a. Out of 35 students in a finite math class, 22 are male, 19 are business majors, 27 are freshmen, 14 are male business students, 17 are male freshmen, 15 are freshmen business majors, and 11 are male freshmen business majors. How many upperclass women non-business majors are in the class? How many women business majors are in the class?

3a. A survey of 100 college faculty who exercise regularly found that 45 jog, 30 swim, 20 cycle, 6 jog and swim. 1 jogs and cycles, 5 swim and cycle, and 1 does all three. How many of the faculty members do not do any of these three activities? How many just jog?

4a. After a genetics experiment, the number of pea plants having certain characteristics were tallied, with the results as follows.

22	were tall
25	produce green peas
39	produce smooth peas
9	are tall and produce green peas
17	are tall and produce smooth peas.
20	produce green peas and smooth peas
6	have all three characteristics
4	have none of the characteristics.

(a) Find the total number of plants counted.
(b) How many plants are tall, but produce peas which are neither smooth nor green?
(c) How many plants are not tall, but produce peas which are smooth and green?

5a. A survey of 80 business executives found the following recommendations on college majors for business students.

36	recommended liberal arts courses
32	recommended business courses.
32	recommended technical courses.
16	recommended technical and business courses.
16	recommended business and liberal arts courses.
14	recommended liberal arts and technical courses
6	recommended all three.

(a) How many executives recommend liberal arts, but neither of the other two?
(b) How many recommend none of these three types of courses?

VENN DIAGRAMS

6a. The musical tastes of a number of college students were surveyed, and it was found that:

22	like Johnny Cash
25	like Elvis Presley
39	like The Carpenters
9	like Johnny Cash and Elvis Presley
17	like Johnny Cash and The Carpenters
20	like Elvis Presley and The Carpenters
6	like all three
4	like none of these performers

(a) How many students were surveyed?
(b) How many like exactly two of these three performers?
(c) How many like Johnny Cash only?
(d) How many do not like Johnny Cash?

7a. The following data shows the preferences of 110 people at a wine-tasting party.

99	like Spanada
96	like Ripple
99	like Boone's Fair Apple Wine
95	like Spanada and Ripple
94	like Ripple and Boone's
96	like Spanada and Boone's
93	like all three.

How many of the people:

(a) like none of the three beverages?
(b) like Spanada, but not Ripple?
(c) do not like Boone's Farm Apple Wine?
(d) like only Ripple?
(e) like exactly two wines?

8a. Routine physical examinations of 500 pre-school children revealed that 40 had dental problems, 45 had vision problems, 55 had hearing problems, 15 had dental and vision problems, 15 had dental and hearing problems, 20 had vision and hearing problems, and 10 had dental, vision, and hearing problems. How many of the children had none of the three kinds of problems?

9a. Human blood can contain either no antigens, the A antigen, the B antigen, or both the A and B antigens. A third antigen, called the Rh antigen, is significant in human reproduction, and again may or may not be present in an individual. Blood is called type A-positive if the subject has the A and Rh, but not the B antigen. Subjects having only the A and B antigens are said to have type AB-negative blood. Subjects having only the Rh antigen have type O-positive blood, etc. In a certain hospital the following data on patients were recorded:

25	patients had the A antigen,
17	had the A and B antigens,
27	had the B antigen,
22	had the B and Rh antigens,
30	had the Rh antigen,
12	had none of the antigens,
16	had the A and Rh antigens,
15	had all three antigens.

(a) How many patients are represented here?
(b) How many patients have exactly one antigen?
(c) How many patients have exactly two antigens?

10a. A local merchant uses television, radio, and newspaper advertising. To determine the effectiveness of advertising, he questions 200 customers during a special after-hours sale to see how they knew about the sale. He found that 115 had seen television ads, 75 had heard radio ads, and 125 had read newspaper ads. He also found that 30 received information from television and radio, 70 from television and newspapers, 25 from radio and newspapers, and 10 from all three. If everyone else said they heard it from a friend, how many heard it from a friend?

11a. In order to prepare a report on agricultural prospects for his county, the county farm advisor questions 100 farmers about their crop plans for the following year. He finds that 75 intend to plant corn, 55 will plant soybeans, 35 will plant wheat, 35 will plant corn and soybeans, 25 will plant corn and wheat, 15 will plant soybeans and wheat, and 10 will plant all three. How many of the farmers will plant only one crop? How many will plant at least two crops?

12a. In a group of 150 primary students, 100 watch “Sesame Street,” 55 watch “Electric Company,” and 65 watch “Mr. Rogers’ Neighborhood.” If, in addition, 35 watch “Sesame Street” and “Electric Company,” 45 watch “Sesame Street” and “Mr. Rogers,” 30 watch “Electric company” and “Mr. Rogers,” and 20 watch all three, how many watch none of the three? How many watch only “Sesame Street”?

VENN DIAGRAMS

13a. At a pow-wow in Arizona, Native Americans from all over the Southwest came to participate in the ceremonies. A coordinator of the pow-wow took a survey and found that:

15	families brought food, costumes, and crafts:
25	families brought food and crafts:
42	families brought food:
20	families brought costumes and food:
6	families brought costumes and crafts, but not food:
4	families brought crafts, but neither food nor costumes:
10	families brought none of the three items:
18	families brought costumes, but not crafts:

(a) How many families were surveyed?

(b) How many families brought costumes?

(c) How many families brought crafts, but not costumes?

(d) How many families did not bring crafts?

(e) How many families brought food or costumes?

14a. A survey of people attending a Lunar New Year celebration in Chinatown yielded the following results:

120	were women:
150	spoke Cantonese:
170	lit firecrackers:
108	of the men spoke Cantonese:
100	of the men did not light firecrackers:
18	of the non-Cantonese-speaking women lit firecrackers:
78	non-Cantonese-speaking men did not light firecrackers:
30	of the women who spoke Cantonese lit firecrackers.

(a) How many attended?

(b) How many of those who attended did not speak Cantonese?

(c) How many women did not light firecrackers?

(d) How many of those who lit firecrackers were Cantonese-speaking men?

VENN DIAGRAMS

15a. A chicken farmer surveyed his flock with the following results. The farmer had:

9	fat red roosters:
2	fat red hens:
37	fat chickens:
26	fat roosters
7	thin brown hens:
18	thin brown roosters:
6	thin red roosters:
6	thin red hens:

Answer the following questions about the flock. [Hint: You need a Venn Diagram with regions for fat, for male (a rooster is a male, a hen is a female), and for red (assume that brown and red are opposites in the chicken world).] How many chickens were:

(a) fat?

(b) red?

(c) male?

(d) fat, but not male?

(e) brown, but not fat?

(f) red and fat?

16. Country-Western songs seem to emphasize three basic themes: love, prison, and trucks. A survey of the local country-western radio station produced the following data:

12	songs were about a truck driver who was in love while in prison:
13	were about a prisoner in love:
28	were about a person in love:
18	were about a truck driver in love.
3	were about a truck driver in prison who was not in love:
2	were about a prisoner who was not in love and did not drive a truck:
8	were about a person who was not in prison, not in love, and did not drive a truck:
16	were about truck drivers who were not in prison.

(a) How many songs were surveyed?

Find the number of songs about:

(b) truck drivers:

(c) prisoners:

(d) truck drivers in prison:

(e) people not in prison:

(f) people not in love:

17A. A survey of 80 sophomores at a western college showed that:

36	took English:
32	took history:
32	took political science:
16	took political science and history:
16	took history and English:
14	took political science and English:
6	took all three

How many students:

(a) took English and neither of the other two?

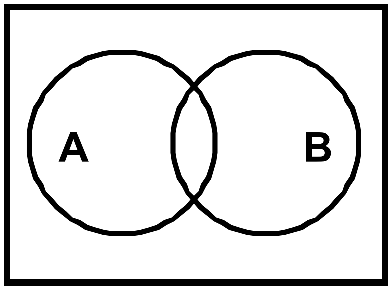
(b) took none of the three courses?

(c) took history, but neither of the other two?

(d) took political science and history, but not English?

(e) did not take political science?

A



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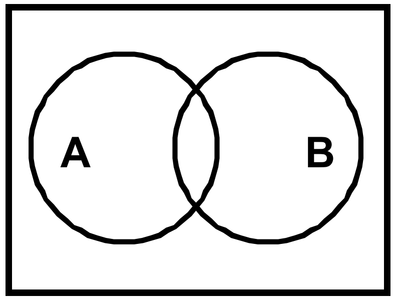

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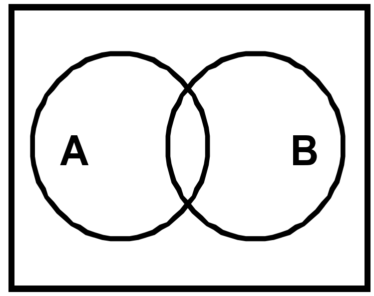

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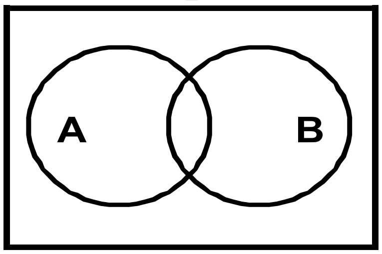

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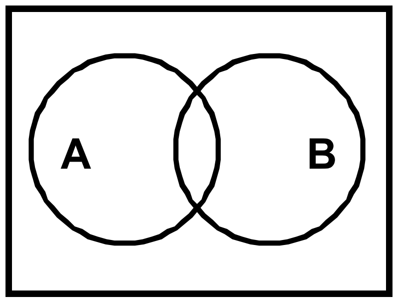

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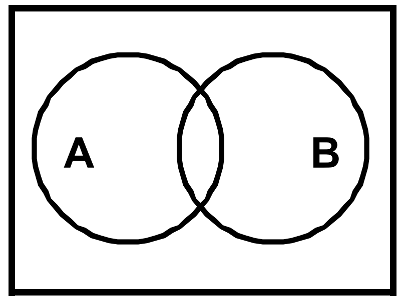

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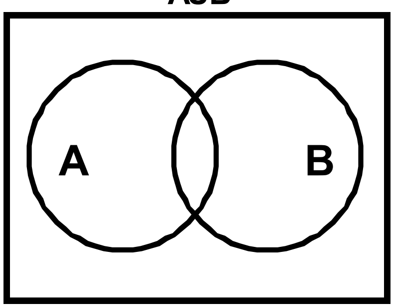

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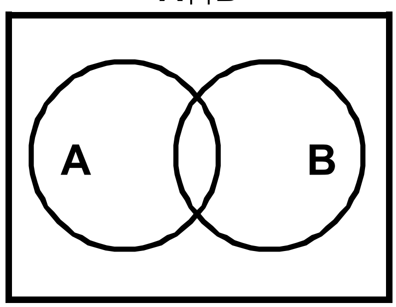

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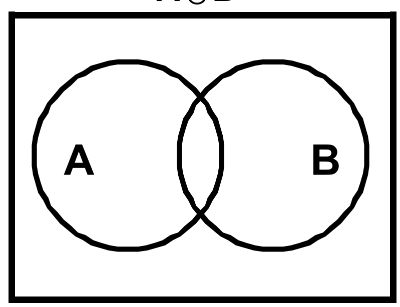

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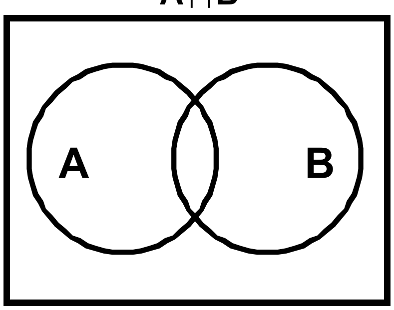

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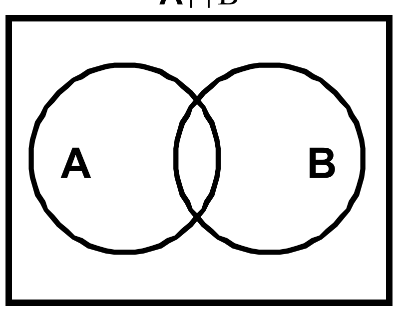

$\overline{A} \cup \overline{B}$