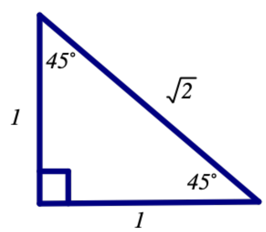
Graphing Trig Functions - Sine & Cosine

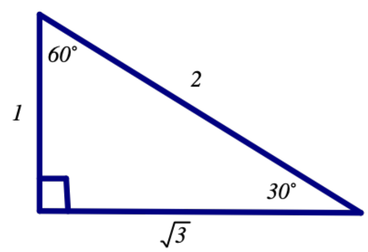
Sec. 1 Graphing the Sine and Cosine Functions

Up to this point, we have learned how the trigonometric ratios have been defined in right triangles. Most of us learned SOHCAHTOA as a memory aid. We then used that information to find the those ratios on a unit circle by using reference angles. Now, we will use that same information to graph trigonometric functions on the Cartesian Coordinate System.

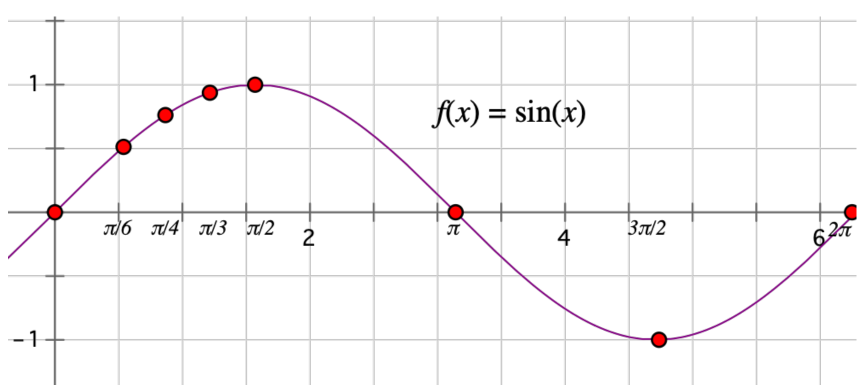
You have to love how all this information ties together. So, let's use the angles in special triangles to help construct my graph and remembering the the x-coordinate is the cosine and the y-coordinate is the sine.

From that information, I know these values in degrees and radians.
sin 0˚ = 0, sin 30˚ = 1/2, sin 45˚ = √2/2, sin 60˚ = √3/2, sin 90˚ = 1
sin 0 = 0, sin π/6 = 1/2, sin π/4 = √2/2, sin π/3 = √3/2, sin π/2 = 1





Example 1 Graph y = sin x for all x such that $0 \leq x \leq 2\pi$



Notice, I labeled my x-axis in terms of radian measure ~ π. Once I relabeled in terms of π, I graphed the first 5 points in the first quadrant ($0 \le x \le \pi/2$). Since the y-coordinates are symmetric in the second quadrant, I can use symmetry to find the graph for $\pi/2 \le x \le \pi$. That gives me the graph in the $0 \le x \le \pi$.

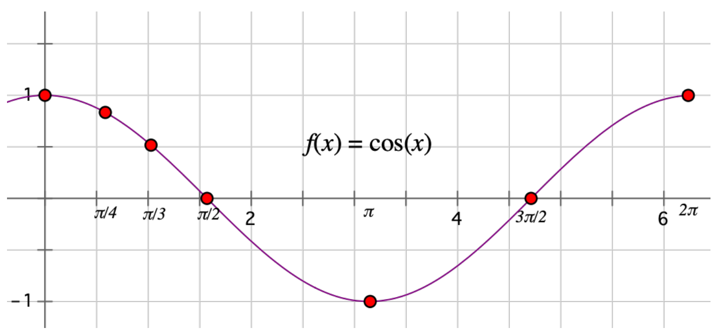
Angles greater than 180 and less than 360˚ fall in the third and fourth quadrants. In the third and fourth quadrants, the ratios are the same but the y-coordinates are negative. In the graph, we can see those points for the special angles fall below the x-axis.

My point, once I label the first few points in the first quadrant, $0 \le x \le \pi/2$, the rest is easy because the only difference in the y- coordinates is the "sign".

Using my special right triangles, lets find the values of the cosine, just like we did for the sine.

$\cos 0 = 1$, $\cos \pi/6 = \sqrt{3}/2$, $\cos \pi/4 = \sqrt{2}/2$, $\cos \pi/3 = 1/2$, $\cos \pi/2 = 0$

Example 2 Graph $y = \cos x$, $0 \le x \le 2\pi$

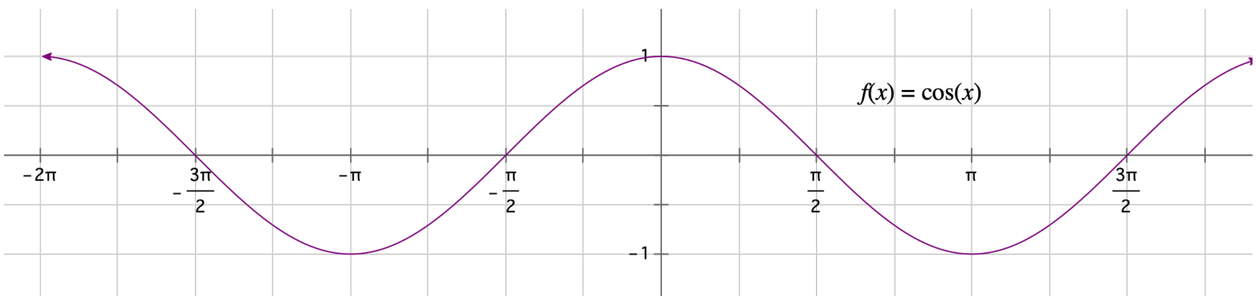


I labeled my x-axis in terms of π and used my special right triangles to find their corresponding values. Again, I only used values in the first quadrant because of symmetry. I know the cosine values (x-coordinates) are negative in the second and third quadrants so the graph should be below the line for $\pi/2 \le x \le 3\pi/2$.

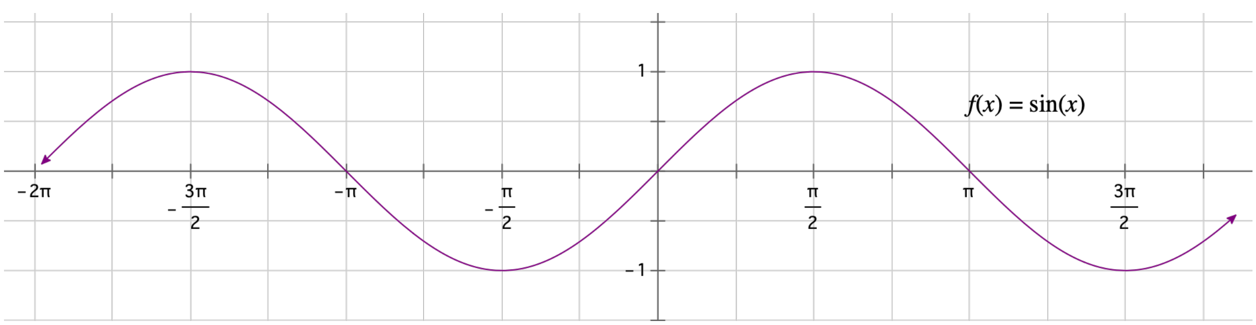
I also know the $\cos \pi = -1$, so I graphed that point, the $\cos 3\pi/2 = 0$, and the $\cos 2\pi = 1$

You might notice that both graphs look like waves. If I started at x = 0 to graph, I see the sine curve's wave goes through the origin. At x = 0, cosine curve's wave passes through (0, 1). If I was able to shift one of the graphs, they would coincide.

Remember when we looked at the unit circle and discussed reference angles. We indicated that an angle whose measure was 405˚ had a reference angle of 45˚. We got that by subtracting 360˚. Let me point out a couple of things, one, since we could make more than one rotation, the graph of the trig functions can go on infinitely, two, we said an angle whose measure is –210˚ has a a reference angle of 30˚ and that suggests the graph goes out infinitely in the negative direction, and three using the coordinates derived from the special right triangles, the waves keep the same size and shape.



So, the graph of the sine and cosine functions are continuous and go out to positive and negative infinity. And, just like on the unit circle, we see the repetition of the values.



Both graphs repeat at multiples of 2π. We say their period is 2π. As we saw on the unit circle and on these graphs, the maximum and minimum heights (y-values) are between one and negative one. We say their amplitude is 1.

Sec. 2 Graph y = $a\sin(bx + c) + d$

And, the really good news, these graphs can be moved around the coordinate system just as we have done with our other graphs; parabolas, circles, absolute value, etc., using the same types of rules and notation.

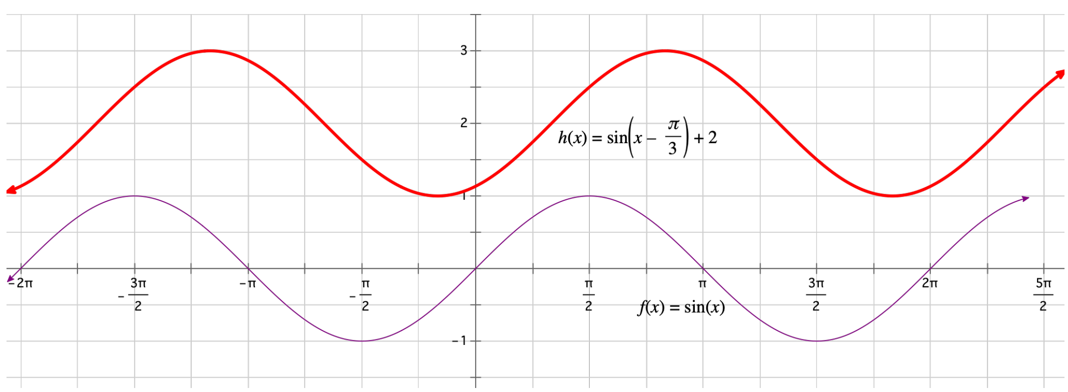
So, recall graphing a parabola, we know what $y = x^2$, looks like. If I changed the equation to $y = x^2 + 1$, the graph of the parabola is moved up one unit. The same is true for sine and cosine graphs, we know what the graph of y = sin x looks like. The graph of y = sin (x) + 1 moves the entire graph up one unit.

To move the graph horizontally, we looked at the value inside the parentheses. So, $y = (x - 2)^2$ moved the parabola over 2 units to the right. Similarly, the graph of y = cos (x – π/2) moves the graph of the cosine over π/2 units to the right.

And finally, if $y = 5x^2$ stretches the graph of the parabola. The same thing occurs with the sine and cosine graphs. That is, y = 5 sin x stretches the sine graph.

So, let's look at an example that translates the graph up 2 and over π/3 and see that graph looks exactly the same but is shifted.

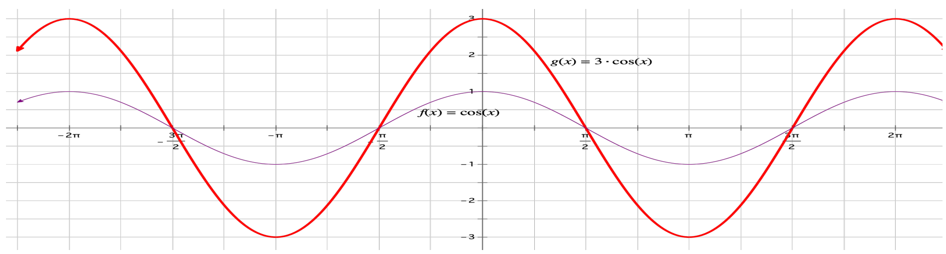
Example 3 Graph y = sin(x – π/3) + 2



Notice the graph of the curve is the same, but all the points were moved up 2 and over π/3.

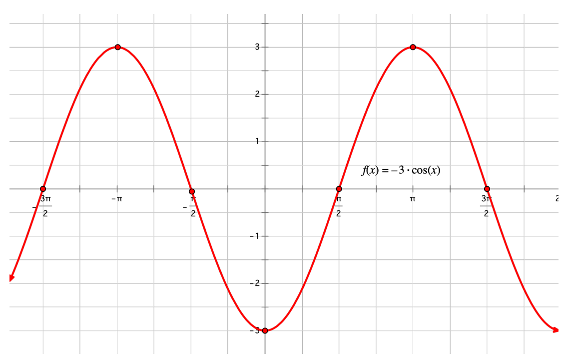
Example Graph y = 3 cos x

In this example, we can still recognize the graph of the cosine, but we can also see it has been vertically stretched by a factor of 3.



What do think might happen to the graph if we multiplied the cosine by negative 3? y = – 3cos x. If you are not sure, try a couple of convenient values of x and see what happens.

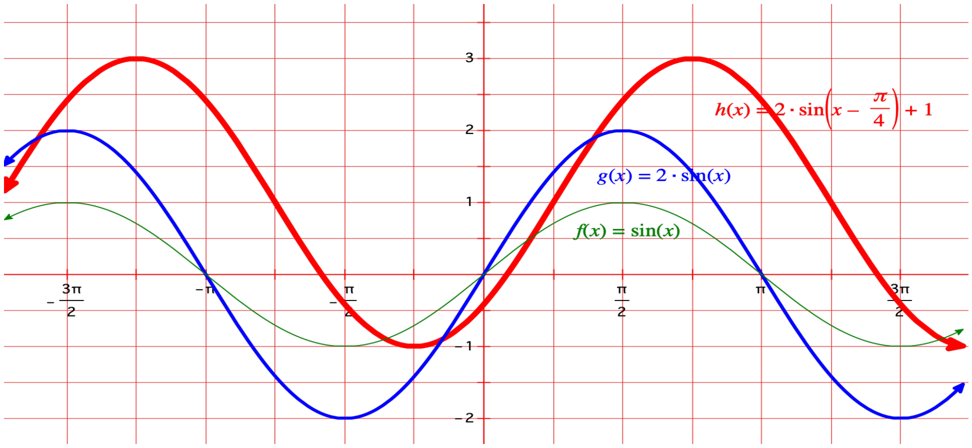
I graphed that by knowing it was going to be a wave that would typically start at (0, 1). But since the cosine was being multiplies by (–3), that my new graph should start at (0, –3). I then recalled my reference angles, at π/2 and 3π/2, x = 0, and at π, x = –1, but I was multiplying the cosine by (–3), so that would result in +3 on the graph. The rest was just symmetry



To graph these trig functions, you should be very aware of what the parent functions look like, then apply the stretch, and horizontal and vertical translations

Let's graph the next equation in pieces so we can see how the added components impact the parent function.

Example 5 Graph $y = 2\sin(x - \pi/4) + 1$



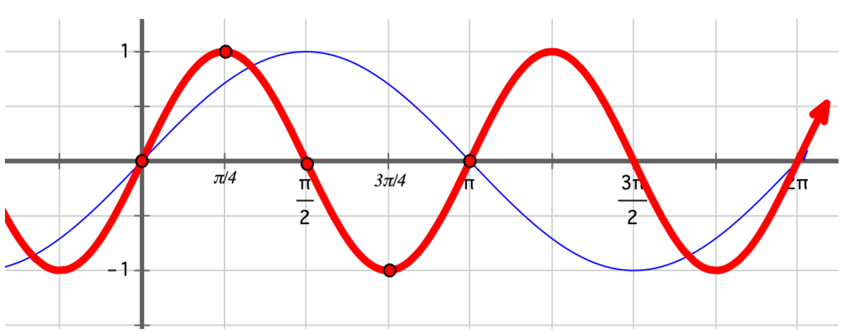
1. Graph the parent function in green, f(x) = sin(x)
2. Graph the vertical stretch, g(x) by multiplying all the y values by 2, the blue graph. Basically making the waves larger
3. Finally moving g(x), blue graph, over π/4 and up 1

Sec. 3 Procedure: Graph y = *a*sin(*b*x + *c*) + *d*

All the waves we studied up to this point repeat every 2π. Can we make them repeat more or less often? Look at the following 2 equations;

f(x) = sin(x) and *h*(x) = sin(2x).

What is physically different in those two equations?



Now, we should be able to sketch f(x) pretty easily. Now, graph h(x) for values of 0, π/4, π/2 and π.

Look at the red graph, what is the period, how often does the wave repeat?

What we have already discovered is if $f(x) = \sin bx$, $b > 0$ ranges between 0 and 2π we get one complete sine wave with amplitude 1. When $b = 1$, the period (one complete sine wave) is 2π. As we can see above, when $b = 2$, $\sin 2x$, we got one complete sine wave at $2\pi/2 = \pi$.

This suggests that the period of a function $f(x) = \boldsymbol{a}\mathbf{sin}(\boldsymbol{b}\mathbf{x} + \boldsymbol{c}) + \boldsymbol{d}$ would be given by $2\pi/b$. This same argument could be used for graphing the cosine and later tangent functions.

And the phase shift (horizontal translation) would be determined by letting $bx + c = 0$ because the graph was defined between $0 \leq x \leq 2\pi$, so bx ranges between $-c$ and $2\pi - c$. That results in $x = -c/b$.

Let's put this together in a procedure.

General Form of Trig Graphs

Graph $y = a\sin(bx + c) + d$;

Procedure:

1. **Identify the midline** $\quad d$
2. **Identify the amplitude** $\quad |a|$
3. **Find the period** $\quad \frac{2\pi}{b}$
4. **Identity the phase shift (bx + c = 0);** $\frac{-c}{b}$

Example 5 Graph $f(x) = 3\sin(2x - \pi/2)$

1. By inspection, the midline is 0,
2. Amplitude is 3
3. The period is $2\pi/b$; $2\pi/2 = \pi$
4. The phase shift is $2x - \pi/2 = 0$, setting $2x = \pi/2$, so the phase shift is $\pi/4$

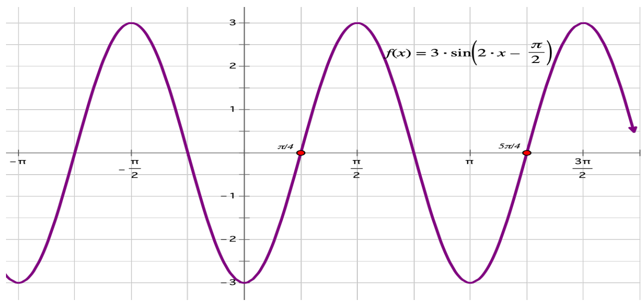
That results in a complete sine wave between $\pi/4$ and $5\pi/4$.

Rather than just using that information, let's walk through our understanding.

To obtain the interval containing the sine wave, we let $2x - \pi/2 = 0$ range from 0 to 2π. Solving, we have

$$2x - \pi/2 = 0 \qquad \text{and} \qquad 2x - \pi/2 = 2\pi$$
$$x = \pi/4 \qquad\qquad\qquad 2x = 5\pi/2$$
$$x = 5\pi/4$$

The amplitude is 3 and the period is $5\pi/4 - \pi/4 = \pi$

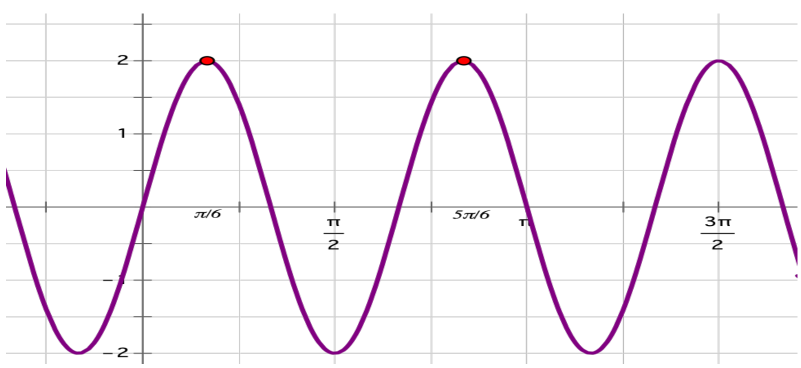


Notice: the one complete period began at π/4 and extended to 5π/4

Example 6 Graph $y = 2 \cos(3x - \pi/2)$

Using our procedure,

1. the midline is 0
2. the amplitude is 2, the
3. period, $3x = 2\pi$ results in $2\pi/3$
4. and the phase shift is $3x - \pi/2 = \pi/6$.



So, from the graph you can see the period went from $\pi/6$ to $5\pi/6$, so the period is $4\pi/6 = 2\pi/3$ and the shift was $\pi/6$ to the right.

Graph the following without plotting points – use the procedure.

1. $y = \sin x$
2. $y = 2 \sin x$
3. $y = \sin 2x$
4. $y = (1/3) \sin (x - \pi/4)$
5. $y = 3 \cos x$
6. $y = -2 \cos 2x$
7. $y = 2 \sin (2x - \pi/3)$
8. $y = 3 \cos (3x - \pi/2)$
9. $y = 2 \sin (2x - \pi/3) + 1$
10. $y = 3 \cos (3x - \pi/2) - 1$