

# DECIMALS

DECIMALS are special fractions whose denominators are powers of 10.

Since decimals are special fractions, then all the rules we have already learned for fractions should work for decimals. The only difference is the denominators for decimals are powers of 10; i.e.,  $10^1, 10^2, 10^3, 10^4$ , etc. ... Students normally think of powers of 10 in standard form; 10, 100, 1000, 10,000.

In a decimal, the numerator is the number to the right of the decimal point. The denominator is not written, but is implied by the number of digits to the right of the decimal point. The number of digits to the right of the decimal point is the same as the number of zeros in 10, 100, 1000, .....

Therefore, one place is tenths, two places is hundredths, three places is thousandths, and so on.

- Examples:**
- 1)  $.56 - 2 \text{ places} - \frac{56}{100}$
  - 2)  $.532 - 3 \text{ places} - \frac{532}{1000}$
  - 3)  $3.2 - 1 \text{ place} - 3\frac{2}{10}$

**The correct way to say a decimal numeral is to:**

- 1) Forget the decimal point.
- 2) Say the number.
- 3) Then say its denominator and add the suffix “ths”

- Examples:**
- 1)  $.53 - \text{Fifty-three hundredths}$
  - 2)  $.702 - \text{Seven hundred two thousandths.}$
  - 3)  $.2 - \text{Two tenths}$
  - 4)  $5.63 - \text{Five and sixty-three hundredths.}$

When there are numbers on both sides of the decimal point, the decimal point is read as “and”. You say the number on the left side, the decimal point is read as “and”, then say the number on the right said with its denominator.

**Example** Write 15.203 in word form  
Fifteen and two hundred three thousandths

## Converting a Decimal to a Fraction

To convert a decimal to a fraction you:

- 1) Determine the denominator by counting the number of digits to the right of the decimal point.
- 2) The numerator is the number to the right of the decimal point.
- 3) Reduce.

### *Example*

- 1) Convert .52 to a fraction.

$$\begin{aligned}.52 &= \frac{52}{100} \\ &= \frac{13}{25}\end{aligned}$$

- 2) Convert .603 to a fraction.

$$.613 = \frac{613}{1000}$$

- 3) Convert 8.32 to a fraction.

$$\begin{aligned}8.32 &= 8\frac{32}{100} \\ &= 8\frac{8}{25}\end{aligned}$$

Try a couple, convert these decimals to fractions.

1.     .23                      2.     .511                      3.     .8                      4.     13.51

## Convert Fractions to Decimals

One way to convert fractions to decimals is by making equivalent fractions.

**Example** Convert  $\frac{1}{2}$  to a decimal.

Since a decimal is a fraction whose denominator is a power of 10, I look for a power of 10 that 2 will divide into evenly.

$$\frac{1}{2} = \frac{5}{10}$$

Since the denominator is 10, I need only one digit to the right of the decimal point, the answer is .5

**Example** Convert  $\frac{3}{4}$  to a decimal

Again, since a decimal is a fraction whose denominator is a power of 10, we look for powers of 10 that that will divide into evenly. 4 won't go into 10, but will go into 100.

$$\frac{3}{4} = \frac{75}{100}$$

There are denominators that will never divide into any power of 10 evenly. Since that happens, we look for an alternative way of converting fractions to decimals. Could you recognize numbers that are not factors of powers of ten? Using your Rules of Divisibility, factors of powers of ten can only have prime factors of 2 or 5. That would mean 12, whose prime factors are 2 and 3 would not be a factor of a power of ten. That means that 12 will never divide into a power of 10. The result of that is a fraction such as  $\frac{5}{12}$  will not terminate – it will be a repeating decimal.

Because not all fractions can be written with a power of 10 as the denominator, we may want to look at another way to convert a fraction to a decimal. That is to divide the numerator by the denominator.

**Example** Convert  $\frac{3}{8}$  to a decimal.

I could do this by equivalent fractions since the only prime factor of 8 is 2. However, we could also do it by division.

$$\begin{array}{r} .375 \\ 8 \overline{)3.000} \end{array}$$

Doing this problem out, we get .375

How do you know how many places to carry out the division? Your teacher would have to tell you tenths, hundredths or thousandths.

1.  $\frac{1}{4}$

2.  $\frac{3}{5}$

3.  $\frac{7}{8}$

4.  $\frac{5}{12}$

Remember, use common sense, if the fraction you are converting is less than one, then the decimal has to be less than one also.

## Converting Repeating Decimals to Fractions

By definition, a repeating decimal is a rational number. That is, it is a decimal that terminates or repeats.

To convert a repeating decimal to a fraction, we will need to use the Properties of Real Numbers.

If I asked you to convert .7 to a decimal, you would simple write 7/10. The same argument would hold for converting .23 to a fraction, that is 23/100.

Now, how is  $\overline{.7}$  different from .7?

How is  $\overline{.23}$  different from .23?

The line over the number (vinculum) is the difference. The vinculum is notation that means the decimal numerals repeat.

So,  $\overline{.7} = .77777\ldots$

$\overline{.23} = .23232323\ldots$

So, as always, we will get rid of what bothers us (the vinculum) by using the Properties of Real Numbers. By letting x equal the repeating decimals, I now have an equation that allows me to use the Properties of Real Numbers.

### My strategy is to convert repeating decimals to fractions

1. Let x = the repeating decimal
2. Multiply both sides of the equation by power(s) of 10 so ONLY the repeating part of the decimal is to the right of the decimal point.
3. Subtract the two equations so the repeating parts subtract out, and
4. Solve the resulting equation which results in a fraction.

Convert  $\overline{.7}$  to a fraction.

$$\text{let } x = \overline{.7}$$

$$10x = 7.\overline{7} \quad \text{multiplying by 10}$$

$$9x = 7 \quad \text{subtracting the two equations, the repeating parts subtract out}$$

$$x = \frac{7}{9}$$

Convert  $\overline{.23}$  to a fraction.

$$\text{let } x = \overline{.23}$$

$$100x = 23.\overline{23} \quad \text{multiplying by 100}$$

$$99x = 23 \quad \text{subtracting the two equations, the repeating parts subtract out}$$

$$x = \frac{23}{99}$$

Convert  $\overline{.123}$  to a fraction.

$$\text{let } x = \overline{.123}$$

$$10x = 1.\overline{23} \quad \text{multiplying by 10}$$

$$100x = 123.\overline{23} \quad \text{multiplying by 100}$$

$$90x = 122 \quad \text{subtracting the two equations, the repeating parts subtract out}$$

$$x = \frac{122}{90}$$

Notice, in the last example, not all the numbers were repeating! To get rid of the vinculum, I multiplied by powers of 10 so ONLY the repeating parts were to the right of the decimal point. We did that so the repeating parts would subtract out.

Convert the following repeating decimals to fractions.

1.  $\overline{.3}$

2.  $\overline{.7}$

3.  $\overline{.5}$

4.  $\overline{.41}$

5.  $\overline{.83}$

6.  $\overline{.21}$

7.  $\overline{.123}$

8.  $\overline{.285}$

9.  $\overline{.371}$

10.  $\overline{.125}$

11.  $\overline{.37}$

12.  $\overline{.35}$

## Comparing Decimals

Since decimals are fractions, you compare decimals the same way you compare fractions. You find a common denominator and make equivalent fractions. The fraction with the largest numerator is the largest fraction.

Now, how does that work with decimals?

**To compare decimal numerals you:**

- 1) Write the decimals so that each decimal numeral has the same number of digits (add zeros) to the right of the decimal point – a common denominator.
- 2) Forget about the decimal points, the largest number will be the largest decimal numeral.

By adding zeros, you are finding a common denominator, just as you did when comparing fractions, now you can compare numerators. Isn't this neat the way this all comes together?

**Examples:** 1) Which is larger .8032 or .82?

Add 2 zeros to .82 so both numbers will have 4 digits to the right of the decimal point.

.8032 and .8200  $\longrightarrow$  both denominators are 10,000

since 8200 is larger than 8032, then  $.82 > .8032$

2) Compare using  $<$ ,  $=$ ,  $>$  .62, .547

Add one zero to .62 so both numbers will have 3 digits. (denominator of 1000)

. 620 and .547

620 is larger than 547, therefore  $.62 > .547$

Use  $>$  or  $<$

1. .9, .235

2. 3.56, 9.1

3. .007, .7

## Adding & Subtracting Decimals

Now, we get to add and subtract decimals. Remember, we defined decimals as fractions. We said all the rules for fractions should then work for decimals. Let's take a look at addition.

If I asked you to add .72 and .126, how would you do it?

Since decimals are fractions, I would use the fraction algorithm.

1. Find the common denominator
2. Make equivalent fractions
3. Add the numerators
4. Bring down the denominator
5. Simplify

In that problem the denominator for .72 is 100, the denominator for .126 is 1000. The **common** denominator then is 1000.

The way to make an equivalent fraction for .72 with a denominator of 1000 is by adding a zero to the end of the number. In other words,  $.72 = .720$ .

Now add the numerators, adding 720 to 126, I get 846. So far, so good.

Now using the fraction algorithm, how do I bring down the denominator of 1000. Well, in order to have a denominator of 1000, I have to have three digits to the right of the decimal point.

So in the number 846, where would I place the decimal point so we have a denominator of 1000? You've got it, before the 8, that would give us .846.

Adding .72 to .126 = .846.

That works because decimals are fractions. But, if we looked at enough addition or subtraction of decimals, we might see a pattern that would allow us to do the problem very quickly using a different algorithm.

### Algorithm for Addition / Subtraction of Decimals

1. Rewrite the problems vertically, lining up the decimal points.
2. Fill in spaces with zeros.
3. Add or subtract the numbers.
4. Bring the decimal point straight down.

By lining up the decimal points and filling in zeros, I have done two things, I have found the common denominator and made equivalent fractions. When I added

the numbers, I added the numerators. And by bringing the decimal point straight down, that accomplishes the same thing as bringing down the denominator. Just as we did with addition and subtraction of fractions. Isn't that neat!

**Example**       $1.23 + .4 + 12.375$

Rewriting vertically

$$\begin{array}{r} 1.23 \\ .4 \\ +12.375 \\ \hline \end{array}$$

Now, filling in the zeros to find the common denominator and make equivalent fractions, then adding we have

$$\begin{array}{r} 1.230 \\ .400 \\ +12.375 \\ \hline 14.005 \end{array}$$

Notice, I brought the decimal point straight down. Bringing down the decimal point is analogous to bringing down the denominator.

Let's try a few of those.

- |                         |                      |
|-------------------------|----------------------|
| 1. $4.23 + .6 + 14.207$ | 2. $9.86 - 4.82$     |
| 3. $15.2 - 6.83$        | 4. $18.2 + 6 + .07$  |
| 5. $13.6 - 5.83$        | 6. $10 + 8.3 - 6.24$ |

Just as the algorithm for adding and subtracting decimals is related to addition and subtraction of fractions, the algorithm for multiplication of decimals also comes directly from the multiplication algorithm for fractions.

Before we see this analogy, let's first see the algorithm for multiplication of decimals.

### Algorithm for Multiplication of Decimals

1. Rewrite the numbers vertically
2. Multiply normally, ignoring the decimal point
3. Count the number of digits to the right of the decimal points
4. Count that same number of places from right to left in the Product (answer)

**Example**       $4.2 \times 1.63$

$$\begin{array}{r} 1.63 \\ \times 4.2 \\ \hline 326 \\ 652 \\ \hline 6846 \end{array}$$

Counting the number of digits to the right of the decimal points, I have two to the right in the multiplicand and one to the right in the multiplier – that's 3 altogether.

Now, we count that same number of places, 3, from right to left in our answer. That's where we put the decimal point.

So our answer is 6.846

Before going on, can you think how this procedure is related to the multiplication algorithm for fractions?

Remember, the algorithm for multiplying fractions,

1. multiply the numerators
2. multiply the denominators
3. simplify

Well, when we multiplied the decimal,  $4.2 \times 1.63$ , what we did by multiplying the numbers without regard to the decimal points was multiply the numerators.

$$4.2 = 4 \frac{2}{10} = \frac{42}{10}$$

$$1.63 = 1 \frac{63}{100} = \frac{163}{100}$$

$$\text{so } 1.63 \times 4.2 = \frac{42}{10} \times \frac{163}{100}$$

Now looking at those two decimals and their fraction equivalents, the denominators are 10 and 100 respectively. If we multiplied the denominators, we would end up with a denominator of 1000.

How many digits do we have to have to the right of the decimal point to have a denominator of 1000? Three zeros, three places.

Guess how many places we move the decimal point when we used the algorithm? You got It – 3.

The point being the algorithm for multiplication of decimals comes from the algorithm from multiplication of fractions. That should almost be expected since decimals are special fractions.

Again, we need to remember that if a number does not have a decimal written, such as 15, the decimal point is understood to go after the number – 15.

Do you want some practice? Sure you do. Try some of these. Besides being able to do these problems, you should know the algorithm and you should understand that decimals are fractions.

- |                      |                        |                       |                       |
|----------------------|------------------------|-----------------------|-----------------------|
| 1. $4.23 \times .6$  | 2. $526 \times .8$     | 3. $4.02 \times .106$ | 4. $.32 \times .09$   |
| 5. $7.23 \times .07$ | 6. $10.01 \times 1.05$ | 7. $73.4 \times .12$  | 8. $725.4 \times 402$ |

## Multiplying by Powers of 10

Let's look at a couple of special cases for multiplication. By looking at a few of these problems, you will be able to multiply in your head. I know that excites you. I'm going to give you a number of multiplication problems just written with their answers. See if you see anything interesting develop?

$$10 \times 12.34 = 123.4$$

$$100 \times 567.234 = 56723.4$$

$$100 \times .0437 = 4.37$$

$$1000 \times 5.678 = 5678.$$

$$10 \times 3.579 = 35.79$$

$$100,000 \times 23.547892 = 2354789.2$$

Looking at the problem and looking at the answer, do you see anything? If you do, that pattern will lead us to another rule.

When you multiply by powers of 10, the product gets larger, so you move the decimal point to the right the same number of places as there are zeros.

**Example**  $10 \times 123.75$

One zero in 10, move the decimal point one place to the right.

Therefore,  $10 \times 123.75 = 1237.5$

**Example**  $100 \times 5.237$

Two zeros in 100, move the decimal point two places to the right.

Therefore,  $100 \times 5.237 = 523.7$

**Example**  $1000 \times 16.2$

Three zeros in 1000, move the decimal point three places to the right. Therefore,  $1000 \times 16.2 = 16200$ . Notice I had to fill in a couple of placeholders to move it three places.

Now you try some.

1.  $8.23 \times 10$
2.  $8.23 \times 100$
3.  $8.23 \times 1000$
4.  $54.2 \times 100$
5.  $100 \times 154.3$
6.  $1000 \times 5.6$
7.  $1.76 \times 10^6$
8.  $8.543 \times 10^2$

Okay, we added, subtracted and multiplied, what do you think comes next?

## Division of Decimals

If you answered, it's what amoebas have to do to multiply, you are so right. Yes, they have to divide. Don't you love a little math humor?

### *Algorithm for Dividing Decimals.*

1. *Move the decimal point as far to the right as possible in the divisor.*
2. *Move the decimal point the same number of places to the right in the dividend.*
3. *Bring up the decimal point straight up into the quotient.*
4. *Divide the way you normally would.*

**Example**  $.31 \overline{)25.834}$

Move the decimal point 2 places to the right in the divisor.

$31 \overline{)2583.4}$

Move the decimal point 2 places to the right in the dividend.

Now, once you have moved the decimal points, you divide normally and bring the decimal point straight up.

$$\begin{array}{r} 83.3 \\ 31 \overline{)2583.4} \\ \underline{248} \phantom{.4} \\ 103 \phantom{.4} \\ \underline{93} \phantom{.4} \\ 104 \phantom{.4} \\ \underline{93} \phantom{.4} \\ 11 \phantom{.4} \end{array}$$

By moving the decimal point the same number of places to the right in the divisor and dividend, what we are essentially doing is multiplying our original expression by **ONE**. In other words, we are making equivalent fractions by multiplying the numerator and denominator by the same number.

Let's look at that

$$\begin{aligned}\frac{25.834}{.31} &= \frac{25.834}{.31} \times \frac{100}{100} \\ &= \frac{2583.4}{31} \\ &= 31 \overline{)2583.4}\end{aligned}$$

So, that is why we are moving the decimal places in the divisor and dividend the same number of places to the right.

If we move the decimal point one place, we are multiplying the numerator and denominator by 10. By moving it two places, we are multiplying the numerator and denominator by 100, etc.

$$\begin{array}{lll}1. & 36 \overline{)194.4} & 2. & 2.4 \overline{).36} & 3. & 4.6 \overline{)1.288} \\4. & .26 \overline{)1302.6} & 5. & 6 \overline{)3.00036} & 6. & .09 \overline{)1.872}\end{array}$$

Again, we should all know fractions and decimals are related. Decimals are special fractions whose denominators are powers of 10. Do you remember looking at special cases for multiplication of decimals? What we are going to do now is look at special cases for dividing decimals.

## Dividing by Powers of 10

Again, what I am going to do is write some problems with their answers. See if you can see a pattern?

### *Examples*

$$23.45 \div 10 = 2.345$$

$$346.853 \div 100 = 3.46853$$

$$87.23 \div 1000 = .08723$$

If we were to look at a few more problems, we might notice the digits in the answer stay the same as the digits in the problem, only the decimal points have moved. Hey, hey, hey, this identifying pattern stuff sure works out nice.

And as always, when we see a pattern that seems to work, we make up a shortcut or rule. Notice when we divide by a power of 10, the number gets smaller so it makes sense to move the decimal point to the LEFT.

When you divide by powers of 10, you move the decimal point to the left the same number of places as there are zeros in the power of 10.

**Example**       $345.8 \div 100$

Since there are two zeros, I move the decimal to the left 2 places. The answer is 3.458.

**Example**       $87.239 \div 1000$

Three zeros, three places, the answer is .087239.

If you forget which way to move the decimal point, use common sense. When you divide by a power of 10, the quotient should get smaller, when you multiply, the product gets bigger.

- |                    |                       |                     |
|--------------------|-----------------------|---------------------|
| 1. $63.5 \div 100$ | 2. $1.874 \div 10$    | 3. $7.12 \div 1000$ |
| 4. $.832 \div 100$ | 5. $2.34 \times 1000$ | 6. $4.56 \div 100$  |

## Scientific Notation

Very large and very small numbers are often written in scientific notation so numbers can be computed easily and as a means of saving space. Even calculators use scientific notation when computing with large or small numbers.

Scientific notation simplifies computing with very large or very small numbers, so it's worth learning.

**To write a number in scientific notation, you rewrite the number as a product of a number between one and ten and some power of ten.**

The next example is contrived. Nobody would use scientific notation to write this number, but it is an easy enough example that will help us see how to rewrite numbers in scientific notation.

### *Example*

Write 420 in scientific notation.

I must rewrite 420 as a product of a number between one and ten and some power of 10.

$$\_\_ \times 10^?$$

Where can I place the decimal point in 420 so it looks like a number between one and 10? Hopefully, you said between the 4 and 2. Let's see what we have.

$$4.20 \times 10^?$$

Since the decimal point is supposed to be to the right of zero, how many places will I have to move the decimal point to get it back to its original position? Two, therefore

$$420 = 4.20 \times 10^2$$

In essence, what we are doing is multiplying by one in terms of the decimal and the power of ten. In the example above, when moving the decimal point 2 places to the left, that is actually dividing by 100, that then requires me to multiply by 100,  $10^2$ , by adding 2 the exponent with base 10.

### *Example*

Write 96,000,000 in scientific notation

Place the decimal point between the 9 and 6, that gives us a number between one and ten

$$9.6 \times 10^? \longrightarrow 96,000,000 = 9.6 \times 10^7$$

To get the decimal point back to its original position, I would have to move the decimal point 7 places to the right, therefore the exponent is 7. Piece of cake!

Write in scientific notation

1. 865,000,000,000

2. 170,000

Let's look at some small numbers.

**Example**

Convert .00000234 to scientific notation

The decimal point goes between the 2 and 3 so we have a number between one and ten. After doing that, how many places do I move the decimal point to get it back to the original position? By counting, we see the decimal point has to be moved 6 places. . .

Since I am moving the decimal point to the right 6 places, the exponent on the 10 will be  $-6$ . Thereby, multiplying by 1. Therefore

$$.00000234 = 2.34 \times 10^{-6}$$

One way to remember if the exponent is positive or negative is that numbers greater than one have positive exponents, numbers less than one have negative exponents.

Write the following in scientific notation.

1. 744,000,000
2. 23,000,000
3. .00027
4. .000000000000000876

Write the following in standard form.

5.  $6.23 \times 10^7$
6.  $5.2 \times 10^{12}$
7.  $3.24 \times 10^{-8}$
8.  $7.4358 \times 10^{-5}$

## Multiplying or Dividing with Scientific Notation

To multiply or divide numbers written in scientific notation is no different from you have already learned. That's what is so great about math!

**Procedure – To multiply or divide numbers written in scientific notation.**

1. Multiply the decimals
2. Multiply the numbers with base 10 by add/sub exponents
3. Rewrite your answer in scientific notation.

**Example**  $(2.3 \times 10^6)(1.2 \times 10^4)$

1. Multiplying the decimals;  $(2.3)(1.2) = 2.76$
2. Multiply the numbers with base 10;  $(10^6)(10^4) = 10^{10}$
3. Write answer in scientific notation;  $2.76 \times 10^{10}$

**Example**  $(4.5 \times 10^3)(3.4 \times 10^5)$

1.  $(4.5)(3.4) = 15.3$
2.  $(10^3)(10^5) = 10^8$
3.  $15.3 \times 10^8$  is not in S.N. the number 15.3 is not between  $1 \leq n < 10$

Therefore, I need to multiply by 1 by dividing 15.3 by 10 and multiplying  $10^8$  by 10.

Final answer is  $1.53 \times 10^9$

## Adding or Subtracting with Scientific Notation

In order to add or subtract numbers in scientific notation, the exponents in the numbers with base 10 have to be the same.

### Procedure – To add or subtract numbers in scientific notation

1. Rewrite numbers with base 10 with the same exponent and move the decimal points with the numbers
2. Add the decimals
3. Write the answer in scientific notation

**Example**  $(1.23 \times 10^5) + (3.4 \times 10^4)$

1.  $(1.23 \times 10^5) + (.34 \times 10^5)$
2.  $1.23 + .34 = 1.57$
3.  $1.57 \times 10^5$

I could have changed the exponent in the first number n that example.

**Example**  $(1.23 \times 10^5) + (3.4 \times 10^4)$

1.  $(12.3 \times 10^4) + (3.4 \times 10^4)$
2.  $12.3 + 3.4 = 15.7$
3.  $15.7 \times 10^4$ ; that's not in S.N.  
 $1.57 \times 10^5$ , dividing & multiplying by 10

Using scientific notation, we can make computation of larger numbers easier.

**Example** Simplify  $\frac{72,000,000 \times 36,000}{180,000}$

$$\frac{7.2 \times 10^7 \times 3.6 \times 10^4}{1.8 \times 10^5} = \frac{7.2 \overset{2}{\cancel{(3.6)}} 10^7 10^4}{\cancel{1.8} \times 10^5}$$

$$= 14.4 \times 10^6 = 1.44 \times 10^7$$

## Rational & Irrational Numbers

A rational number is a number that can be written in the form  $\frac{a}{b}$ . Decimals are rational if they terminate or repeat because they can be written as a fraction. The following are examples of rational numbers.

$$8, \frac{3}{4}, 43, \bar{3}$$

An irrational number cannot be written in the form  $\frac{a}{b}$ . Decimals are irrational if they are nonterminating, and nonrepeating which cannot be written as a fraction. The following are examples of irrational numbers.

$$\pi, e, \sqrt{5}, .313313331\dots$$

## Estimating Square Roots

To simplify square roots quickly, you should be familiar with perfect squares. You find perfect squares by listing the Counting Numbers (1, 2, 3, ...) and squaring them.

### *Perfect Squares*

$$1^2 = 1$$

$$2^2 = 4$$

$$3^2 = 9$$

$$4^2 = 16$$

$$5^2 = 25$$

$$6^2 = 36$$

$$7^2 = 49$$

$$8^2 = 64$$

$$9^2 = 81$$

$$10^2 = 100$$

The numbers on the right-hand side of the equal signs are called **perfect squares** because they come from squaring a number.

If I asked you to find the  $\sqrt{25}$ , my guess is you would know the answer is 5. In fact, if I asked you to find the square root of numbers like 16, 25, 36, 49, or 100 – perfect squares – you would probably know those answers as well because you know your multiplication facts. Why? Because they are perfect squares.

What if you are asked to find the square root of a number that is not a perfect square? The short answer is just don't know. But, you could approximate it. What's an approximation of the  $\sqrt{50}$ ?

Since 50 is not a perfect square, we can look at our table of perfect squares above and see that 50 falls between the perfect squares of 49 and 64, so the  $\sqrt{50}$  should be between 7 and 8. So an approximation might be 7.5. But looking at the table, we can also see that 50 is a lot closer to 49 than 64 and estimate the  $\sqrt{50}$  might be closer to 7.2. If we squared 7.2, the result is 51.84. That's greater than 50, so our answer now lies between 7 and 7.2. We could continue this process to get closer and closer to the  $\sqrt{50}$ .

## Simplifying Radicals (Square Roots)

What would happen if I asked you to simply  $\sqrt{50}$  so we have an exact answer, not an estimation? My guess is you would run into some trouble. The fact is you don't know the  $\sqrt{50}$ . Is there any way to simplify that mathematically? You bet, otherwise I would not have brought it up.

**To simplify a square root:**

1. Rewrite the radicand as a product of a perfect square and some other number.
2. Take the square root of the perfect square.
3. Leave the other number in the radical.

**Example** Simplify the  $\sqrt{50}$

$$\begin{aligned}\sqrt{50} &= \sqrt{25 \times 2} \\ &= 5\sqrt{2}\end{aligned}$$

Now, if I asked you to simplify the  $\sqrt{300}$ , you realize it was not a perfect square. So you would rewrite 300 as a product of a perfect square and some other number. Look at the table of perfect squares, which is the greatest factor of 300? Hopefully, you said 100. Therefore, we have

$$\begin{aligned}\sqrt{300} &= \sqrt{100 \times 3} \\ &= 10\sqrt{3}\end{aligned}$$

Estimate and simplify the following square roots.

1.  $\sqrt{20}$

2.  $\sqrt{32}$

3.  $\sqrt{72}$

4.  $\sqrt{45}$

5.  $\sqrt{98}$

## Definitions

1.\*\*\* Decimal

2.\*\*\* Rational Number

3.\*\*\* Irrational Number

4.\*\*\* Procedure to convert a fraction to a decimal

5.\*\*\* Write the algorithm for multiplying decimals.

6.\*\* Write 16.237 in word form.

7.\*\* Write 32.405 in expanded notation

8\* How are the algorithms for ADD/SUB fractions and decimals related?

9.\* When dividing decimals, why do you move the decimal point in the dividend the same number of places to the right as you move it in the divisor?

10.\*\* Express in scientific notation

a. 645,000,000,000

b. .0000000468

11.\*\*  $12.345 + 8 + 2.03$

12.\*\*  $13 - 5.67$

13.\*\*  $102.3 \times 6.2$

14.\*\*  $9.1 \overline{)20.762}$

15.\*\* Use  $16 \times 124 = 1984$  to find  $1.6 \times 1.24$

16.SBAC Choose all the correct equations:

a.  $10 + .7 = 17$

b.  $.7 + .3 = 1$

c.  $10 \times .7 = 7$

d.  $.8 + .2 = 10$

17.\*\* Convert to a fraction

a) .053

b) .9

18.\*\* Convert to a fraction

$\overline{.23}$

19.\*\* Convert to a decimal

a)  $\frac{3}{8}$

b)  $\frac{1}{9}$

20.\*\* Round 54.3746 to the nearest thousandth.

21.\*\* Arrange the from least to greatest.

.251, .25,  $\frac{1}{5}$ , .1004

22.\*\* Simplify

$$\frac{(4.2 \times 10^7)(8.1 \times 10^4)}{2.1 \times 10^3}$$

23.\*\* Simplify

a)  $\sqrt{36}$

b)  $\sqrt{300}$

24.\* Lenny went to the store and bought a chair for \$17.95, a rake for \$13.59, a spade for \$14.84, a lawn mower for \$189.99, and two bags of fertilizer for \$3.29 each. If the state tax is 6%, what was his bill?

25.\* SBAC Circle all the rational numbers:

$$\sqrt{2}, \quad \frac{3}{7}, \quad \pi, \quad .23, \quad .1\bar{7}, \quad .2346..., \quad \sqrt{25}$$

26.\*\*\*Provide parent/guardian contact information: phone, cell, email etc.  
(CHP)