WORD Problems

1. **Using a periodic function, model a person's height on a ferris wheel using y = sin x where x measures time in seconds and y measures the height above ground.**

If rider on a Ferris wheel starts at the **midpoint height** of 50 feet and rises to a maximum of 90 feet and the wheel completes one rotation every 60 seconds. Write an equation to model a rider's height.

This is a periodic function because its repeating, round and round, Both the sine and cosine functions are periodic – they repeat. In this problem the person is starting at 50 feet and goes to a height of 90 feet. That means they go as low as 10 feet. S

Since they are starting at 50 feet and going around from there, 50 is the midline, the amplitude, going up 40 feet is the amplitude. The wheel goes around every 60 seconds, the period is defined as $2\pi/B$, which means $B = \pi/3$. To determine to use the sine or cosine, we look where the ride starts, it starts at the midline, so we use the sine function and substitute the numbers.

Amplitude = $\frac{90-10}{2} = 40$
Midline = 50
Period = 60 → $B = \frac{2\pi}{60} = \frac{\pi}{30}$

$$y = \boldsymbol{a} \sin Bx + \boldsymbol{d}$$

$$h(t) = 40 \sin \left(\frac{\pi}{3} t\right) + 50$$

2. **At a beach, the water level is highest at 8 feet and lowest at 2 feet. The time between two high tides is 12 hours. Write an equation to model the water height.**

Since we are starting at high tide, a maximum height, we will use the cosine function. The midline (midpoint) is given by (8+2)/2 = 5, and goes up and down 3 feet, the amplitude is 3. It repeats every 12 hours, so the period is 3 resulting $B = \pi/6$

Amplitude = $\frac{8-2}{2} = 3$
Midline = 5
Period = 12 → $B = \frac{2\pi}{12} = \frac{\pi}{6}$

$$h(t) = 3\cos\left(\frac{\pi}{6}t\right) + 5$$

3. **The temperature varies from 60°F to 80°F over a 24-hour period. At midnight, the temperature is at the average and increasing. Write an equation to model the change in temperature.**

The average temperature, the midline, occurs at midnight and is increase. That suggests the sine function. Since the temperature range from 60˚ to 80˚, the midline is 70˚. The amplitude then is 10.

Amplitude = $\frac{80-60}{2} = 10$
Midline = 70
Period = 24 → $B = \frac{2\pi}{24} = \frac{\pi}{12}$

Filling in the information into the equations, we have: T(t) = 10 sin $(\frac{\pi}{12}t)$ + 70

4. **A pendulum swings with a maximum displacement of 5 cm from the center. It completes one full swing every 4 seconds. Write an equation for displacement.**

Since we begin the swing of the pendulum from 5 feet from the center, we are not starting at the midline, so we will use y = cos x. One complete swings takes 4 seconds, so the period is 4.

Amplitude = 5
Midline = 0
Period = 4 → $B = \frac{2\pi}{4} = \frac{\pi}{2}$

Filling the information into the cosine equation, we have D(t) = 5 cos $\frac{\pi}{2}$ t

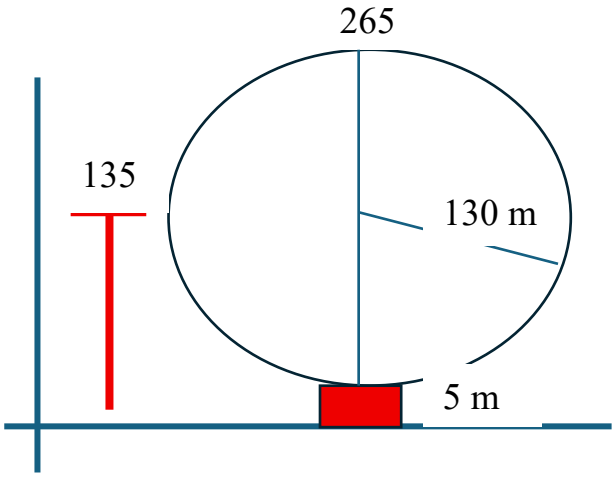
5. **A rotating beacon light varies in intensity from 0 to 100 units. It completes a full rotation every 10 seconds. At time $t = 0$, the intensity is at the midpoint and increasing. Write an equation for intensity.**

Amplitude = 50
Midline = 50
Period = 10 → $B = \frac{2\pi}{10} = \frac{\pi}{5}$

$$I(t) = 50 \sin\left(\frac{\pi}{5}t\right) + 50$$

6. **A ferris wheel rotates every 30 minutes. If the diameter of the wheel is 260 meters and a person boards from a platform 5 feet above round, write an equation for height.**

It might be helpful to draw a picture. The period is 30 minutes, so B = 2π/30 = π/15. The amplitude, the radius is 130 meters and we are starting at the bottom of the ferris wheel to get on, that suggests the cosine curve. Here's where the picture will help; if the wheel start at ground level, we would be going from zero to a height of 260 meters with a midline of 130 meters. But, we are starting from a platform 5 feet above ground, so our midline is 135 meters and our radius, the amplitude, is 130 meters



Since we are starting at the bottom and going up, we will use the negative cosine.

Amplitude is 130
Period is 30 minutes, so B = π/15
Midline is 135

$$h(t) = -130 \cos\left(\frac{\pi}{15} t\right) + 135$$

Now, using that equation, like in the other problems, I could ask a question that you would use the equation. Such as, how high off the ground would you be in 10 minutes? Of course, you would substitute 10 for t in your equation.