Understanding Math

Southern Nevada Regional Professional Development

Proportional Reasoning

In this issue of *Understanding Math* we close our discussion of proportional reasoning and the errors in language that relate to it. In the March 2005 edition, we focused on *absolute* differences in percentage as compared to *relative* differences. This time, we'll extend this to what one might call "errors of symmetry" in the use of percentage differences.

Imagine a company that produced 8 units of some product last year. This year, the company produced 10 units of the product. By what percentage did the company increase its output? As we saw last month, $\textit{relative}\,\text{change} = \dfrac{\text{final condition} - \text{initial condition}}{\text{initial condition}} = \dfrac{\underline{\textit{absolute}\,\text{change}}}{\text{initial condition}}$. So,

$\textit{relative}\,\text{change} = \dfrac{10-8}{8} = \dfrac{2}{8} = 0.25 = 25\%$. The company's output grew by one-fourth or 25 percent.

Now, imagine a similar company that produced 10 units last year and only 8 units this year. What was the percentage decrease in production? Many students would quickly reply, "Twenty-five percent." After all, if going from 8 to 10 is an increase of 25%, then going from 10 to 8 should be a decrease of 25%, right? No. This is an error of symmetry. Let's do the math:

$\textit{relative}\,\text{change} = \dfrac{\text{final condition} - \text{initial condition}}{\text{initial condition}} = \dfrac{8-10}{10} = \dfrac{-2}{10} = -20\%$. The absolute change is 2 units, and that change is based on 10 units, not 8 units. Two is 20% of 10, not 8.

Errors of symmetry can also be shown by this classic math problem: *An employer announces to his staff that because of poor sales, salaries will be cut by 10%. Later, when sales improve, the staff will receive a 10% pay raise to offset the cut. What will an employee be making after the cut and the subsequent raise?* Once again, students will be inclined to state that employees will be making the same salary that they started with. After all, a cut of 10% followed by a raise of 10% balances out. This is an error of symmetry when dealing with *relative change*.

Let's take an employee making \$100/day. After a 10% pay cut, the worker would drop by \$10/day to \$90/day. But a 10% raise on that \$90/day adds only \$9/day to the salary, resulting in \$99/day. The net result is that the worker ends up with an overall pay cut of 1%.

This can (and should) be shown algebraically. Let the employee's original salary be S. Ten percent of the salary is $0.10S$, so the cut results in a new salary of $S - 0.1S = 0.90S$. Now, ten percent of $0.90S$ is $0.09S$. After the raise, the final salary is $0.90S + 0.09S = 0.99S$, which is 99% of S. The employer isn't doing the worker any favors.

The errors of symmetry discussed above pertain to *relative* change, not *absolute* change. If the employer had cut salary by 10 *dollars* rather than ten percent, then raised it 10 dollars, the outcome would be a wash. Essentially, we would have $S - \$10 + \$10 = S$. In absolute terms, equal gains and losses cancel. In the case of the company growth/loss example, both companies changed *absolutely* by two units.

The moral of the story is that students need to communicate whether any differences to which they refer are *absolute* or *relative*.

bill@hanlonmath.com