Algebra, Recursive Functions

Been there – Done that

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Algebra, Been there –Done that is a newsletter that links algebra to previously learned concepts and skills or outside experiences

A linear function can describe an arithmetic sequence by substituting values of x beginning with one and finding the corresponding values of y.

An arithmetic sequence is linear because there is a constant rate of change

Example **f(x) = 2x + 3**

x	1	2	3	4	5
y	5	7	9	11	13

The resulting sequence is 5, 7, 9, 11, 13, …

Common Difference
Rate of Change
Slope

Notice the slope is 2 – the common difference is 2.

To find the third term of the sequence, we can see from the chart it is 9. Or, we could find the value of f at 3, written mathematically as f(**3**) = **9**.

The x-y chart could be written as ordered pairs; (1, 5), (2, 7), (3, 9), (4, 11), …

Look at the chart, look at the sequence, and look how the f is defined. Now look at the next three statements.

Note the 3rd term is found by adding 2 to the 2nd term. In other words f(3) = f(2) + 2
Note the 4th term is found by adding 2 to the 3rd term. In other words f(4) = f(3) + 2
Note the 5th term is found by adding 2 to the 4th term. In other words f(5) = f(4) + 2

You can use the Point-Slope Form of a Line to find an equation

I'm adding 2 because that is the common difference, the rate of change, the slope. Also note, I am adding 2 to the preceding function. That is, f(5) is being described in terms of f(4).

$$y - y_1 = m(x - x_1)$$

Generalizing, to find the function recursively, I'm merely adding the common difference (which is the slope) to the preceding term in the sequence which is described in functional form.

Now, to ensure we understand the notation, let's look at **f(5) = f(4) + 2**.
Another way to write that is **f(5) = f(5 – 1) + 2**

In general

So, to write the function **recursively**, we write the nth term in term in terms of the preceding term; (n – 1).

$$f(x) = f(x - 1) + d$$

Mathematically, we say that **f(n) = f(n– 1) + d**, where d is the common difference or slope.

Example If g(x) = 5x + 3, write g(x) in terms of g(x – 1)

What's the slope? It is 5, so **g(x) = g(x – 1) + 5**

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Example If h(x) = –2x + 10, write h(x) in terms of h(x – 1)

What's the slope? It is –2, so **h(x) = h(x – 1) – 2**