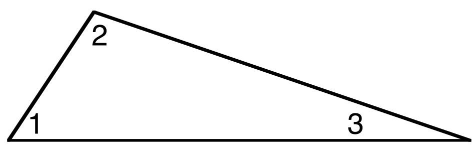
Geometry, *You Can Do It !*
Angle Theorems: Polygons

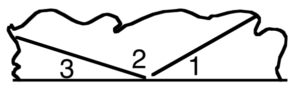
by Bill Hanlon

If I asked an entire class to draw a triangle on a piece of paper, then had each person cut out their triangle, we might see something interesting happen.

Let's label the angles 1, 2 , and 3 as shown.



By tearing each angle from the triangle, then placing them side by side, the angles form a straight line. Neato !



That might lead me to believe the sum of the interior angles of a triangle is 180°.

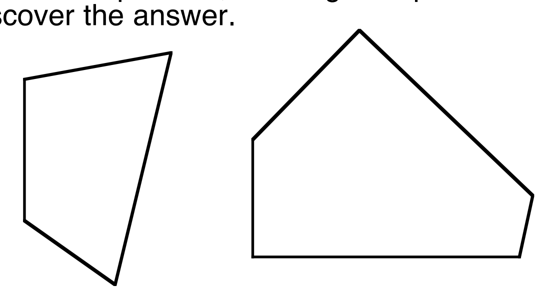
While that's not a proof, it does provide me with some valuable insights. The fact is, it turns out to be true, so we write it as a theorem.

Theorem

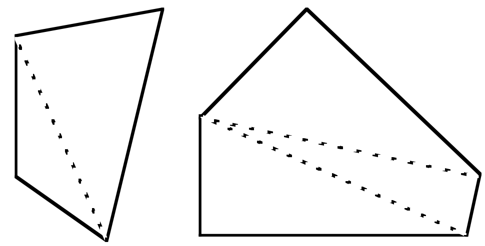
The sum of the interior angles of a triangle is 180°.

Will that help us find the sum of the interior angles of any convex polygon? What 's the sum of the interior angles of a quadrilateral? a pentagon? an octagon?

The answer is, I just don't know. But ..., if I draw some pictures, that might help me discover the answer.



By drawing diagonals from a single vertex, I can form triangles.



4 sides 5 sides

2 Δ's 3 Δ's

That might lead me to believe the sum of the interior angles of a quadrilateral is 360°.

In a five sided figure, a pentagon, three triangles are being formed. Three times 180° is 540°.

The number of triangles being formed seems to be two less than the number of sides in the polygon. Try drawing an octagon and see if the number of triangles formed is two less than the number of sides.

So a polygon with n sides would have (n–2) triangles formed. So, If I multiply the number of triangles by 180°, that should give me the sum of the interior angles. Sounds like a theorem to me.

Theorem

The sum of the interior angles of a convex polygon is given by

$$(n-2)\ 180°$$

Using that, and since a hexagon has six sides, the sum of the interior angles should be (6–2) times 180° or 720°.

If we played with these pictures longer, we'd find more good news in the form of a theorem.

Theorem

The sum of the exterior angles of a convex polygon, one angle at each vertex, is 360°.

