

Geometry, You Can Do It !

Area - circles

by Bill Hanlon

To find the area formula of a circle, we will use the same techniques we used to find areas of other polygonal regions. That is we will make it look like something we already have studied.

Since a circle does not have sides, it is not a polygon. So the distance around it is not called perimeter, it's called a **circumference**.

Interestingly, it turns out that the circumference of any circle divided by the diameter always equals 3.1415926545....

$$\frac{C}{d} = 3.1415926545....$$

Since 3.1415926545... is hard to remember and it occurs frequently, we'll give that number a special name, π .

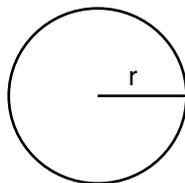
That means we can rewrite that formula as

$$\frac{C}{d} = \pi$$

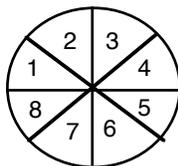
An equivalent expression is $C = \pi d$. Since a diameter has the same measure as two radii, we can also write the circumference formula in terms of the radius.

$$C = 2 \pi r$$

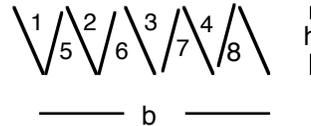
With that information, we can now try to find a way to determine the area of a circle with radius r



To do that, I will break the circle up in eighths.



Now, I'm going to cut out those sections and rearrange them as shown.



I need you to notice a couple of things before we move forward. Note from the circle, sections 1,2,3,and 4 compose half the distance around the circle. That means that is equal to half the circumference. Also note that the rearrangement begins to look like a parallelogram. It would look more and more like a parallelogram if I continued to break up the circle into more sections.

Remember, the area of a parallelogram is given by the formula bh . The base in our rearrangement is composed of sections 5,6,7, and 8, which is half the circumference or πr . The height of the parallelogram is the radius.

Using the formula for parallelogram, we have

$$A = b h$$

Substituting πr for b and r for h , we have

$$A = \pi r r$$

OR

$$A = \pi r^2$$

That means the area of the 8 sections is πr^2 . Putting those sections back together, we have the area of a circle.

Theorem

The area of a circle is equal to the product of π and the square of the radius of the circle.

$$A = \pi r^2.$$

Because we are measuring area, the answer is always written in square measure.